Dynamic Phosphorus and Nitrogen Yield Response Model for Economic Optimisation

Matti Sihvonen [a,1], Kari Hyytiäinen [a], Elena Valkama [b], Eila Turtola [b]

[a] University of Helsinki, Finland, Department of Economics, Latokartanonkaari 7, 00790 Helsinki, Finland
[b] Natural Resources Institute Finland (LUKE), Management and Production of Renewable Resources/ Environmental impact, Tietotie 4, Jokioinen, FI-31600, Finland
Conflicts of interest: none

Abstract

This paper provides an approach for modelling joint impact of two main nutrients in crop production for situations where there are available separate datasets for nitrogen and phosphorus fertiliser field experiments. Developing yield response models for Finnish spring barley crops (*Hordeum vulgare L.*) for clay and coarse soils and applying the models for dynamic economic analysis demonstrate the modelling approach. Model selection is based on iterative elimination from a wide diversity of plausible model formulations. Nonlinear weighted least squares method was utilised in estimation of the yield response models and dynamic programming was utilised in economic analysis. Our results suggest that fertiliser recommendations can be insufficient if soil phosphorus dynamics are ignored. Further, the optimal fertilisation rates for nitrogen and phosphorus, as well as the economic alternative costs of agri-environmental programmes depend on the soil texture of production area. Therefore, the efficiency of such programmes could be improved by targeting different fertilisation limits for different soil textures. In addition, uncertainty analysis revealed that the parameter uncertainty had a greater effect on the model output than the structural uncertainty. Further, the interaction of nitrogen and phosphorus fertilisers appeared to be a factor of relatively minor importance. The modelling approach and the model structure can be extended to other geographical areas, given that adequate datasets are available.

Keywords: [phosphorus fertilisation; nitrogen fertilisation; plant-available soil phosphorus; yield response model; model selection; dynamic optimisation][2]

1 Introduction

The motivation for more efficient use of nutrient inputs in agriculture has never been greater than it is at present (Roberts, 2008). This motivation originates from the need to meet increasing global food demand under finite primary fertiliser sources (Gilbert, 2009), finite arable land, expected increase in energy cost (Snyder, 2006), and growing public concerns related to the environmental effects of agriculture (Reetz, 2016). In addition, climate change is expected to affect these issues in multiple

[1] Corresponding author: matti.sihvonen@helsinki.fi. Tel.:+358 45 6343815. Huhmarekuja 4 A 14, 00790 Helsinki

[2] **Abbreviations:** [STP: Soil test phosphorus, AIC: Akaike's information criteria, DSA: deterministic sensitivity analysis, FAEP: Finnish agri-environmental programme]

uncertain, complex, and possibly controversial ways (e.g., Kang et al., 2009; Gornall et al., 2010; Popp et al., 2017).

Mineral phosphorus (P) and nitrogen (N) inputs are critical both for the profitability of crop cultivation and its externalities. N and P losses from agricultural soils to water and air are a cost for the farmer and may deteriorate the quality of surface water and groundwater as well as cause greenhouse gas emissions. In aquatic ecosystems, over-enrichment of N and P may increase toxic algal blooms and oxygen depletion, and alter the structure and functioning of aquatic food webs and biodiversity (including stocks of species important to commercial fisheries and shellfish industries) (e.g., Sharpley et al., 1994; Alley et al., 2009). One of the fundamental causes of the N and P losses from an agricultural production area is an inefficient fertilisation utilisation.

Bio-economic models can be utilised as tools to study economically and environmentally sound fertiliser inputs in crop production. Empirical models have been developed to capture the yield response to N fertilisation in multiple studies (e.g., Bock and Sikora, 1990; Cerrato and Blackmere, 1990; Valkama et al., 2013). Analogously, yield response to P fertilisation have been studied by e.g. Judel et al. (1985), Saarela et al. (1995), Valkama et al. (2011). In addition, there have been reports focusing on yield response to plant-available soil phosphorus (e.g., Yajragupta et al., 1963; Analogide and Rendig, 1972; Dodd and Mallarino, 2005). Nevertheless, in these studies, the joint response to all of these inputs was not examined. Both nutrients (N and P) have been applied simultaneously as inputs in stochastic production frontiers (see e.g., Bäckman and Lansik, 2005), which are commonly used to examine the nutrient productivity and efficiency differences between different agro-economic conditions (Aigner et al., 1977; Meeusen and van den Broeck, 1977). The joint impact of N and P inputs on crop yield is also explicitly considered in dynamic crop growth simulation models, such as DAISY (Hansen et al., 1990), HYPE (Eckersten et al., 1994), APSIM (Keating et al., 2003) and EPIC (Strauss et al., 2012; van der Velde et al., 2014). Although these models give a rich and detailed description of the processes that drive the yield response, they are not directly suited to dynamic economic optimisation or stochastic analysis due to large number of state variables.

To the best of our knowledge there appears to be no yield response models directly applicable for the analysis of optimal fertilisation decisions, which would capture the joint impacts and dynamic feedbacks of several main nutrients. The obvious reason for this is the need for extensive, long-term datasets from field experiments to capture the weather-induced variation in the impacts of N and P inputs and soil qualities on crop growth (Bolland et al., 2003). Such field experiments are expensive to conduct (Xiaofei et al., 2016). Yield response modelling is also a data-intensive process because data are needed for both building and validating the model (Sargent, 2011).

In addition, for practical applications, data is needed also for model calibration (Rötter et al., 2012). In Finland, there is a long tradition of N and P fertiliser field experiments, motivated by the challenging agro-climatic production conditions resulting from a Nordic location and natural scarcity of P in the soil (Saarela et al., 1995, 2004; Valkama et al., 2009, 2011, 2013; Salo et al., 2013). As a result of these experiments, extensive sets of empirical data have been accumulated over five decades for the main cereals and different soil textures.

In this study, datasets for spring barley and two soil texture groups, clays soils and coarse-textured mineral soils, were combined and utilised in order to develop a yield response models to both P and N inputs applied simultaneously. In addition to N and P, the plant-available soil test phosphorus (STP) and initial productivity of the soil (the yield without added N fertilisation) were included in the model as independent variables, since these factors govern the yield responses to P and N fertilisation, respectively (Valkama et al., 2011, 2013). The inclusion of STP enables the examination of dynamic aspects of optimal agricultural practises, when the model is coupled with a compatible transition model describing the soil P dynamics. Our objective was to develop system models that can be directly applied in dynamic bio-economic analysis and to assess the economic and environmental consequences of mineral fertilisation in crop cultivation. In particular, we were interested in: (1) the dynamic interactions of the yield responses and the soil nutrient transition and their impact on economic outcomes, (2) examining the diversity of the functional forms that can be fit to the empirical fertilisation experiment data, (3) the related model structural and parametric uncertainty, (4) the robustness of the predictive and optimisation results for various candidate models, (5) to assess the economic optimums and their comparison to current national fertilisation recommendations and (6) to evaluate the generalisability of the models.

2 Materials and methods

2.1. Structure of the modelling process

We applied an iterative elimination approach for the modelling process. We began by confining the essential dynamic feedbacks of the agricultural system under examination. Second, we gathered the datasets for the estimation process. Third, we estimated and ranked the yield response models. Fourth, we combined the individual model elements to integrated models and evaluated their performance first via simulations and second via validating the models with additional datasets gathered for the study. Fifth, we coupled the yield response models with the transition models and applied the system models for dynamic economic optimisation; the robustness of the results was evaluated and the results were compared to the maximum permissible N and P rates denoted by the Finnish agri-environmental programme (FAEP). Last, we evaluated the effects of structural and parametric uncertainty on economic optimums via uncertainty and sensitivity analysis. The iterative elimination modelling process is shown in Figure 1.

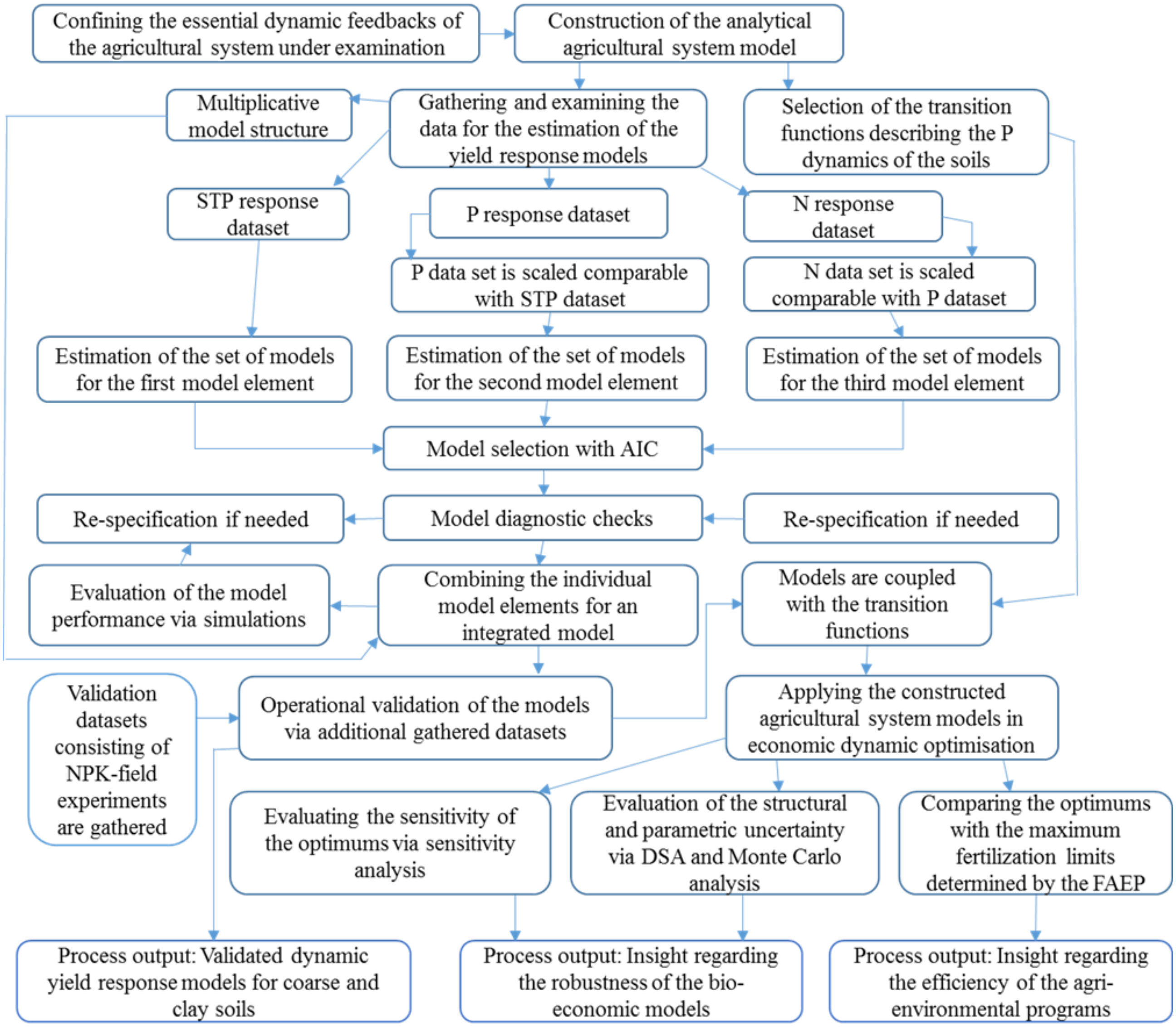


Fig. 1. Schematic diagram of the iterative elimination modelling process carried out within the study. AIC stand for Akaike's information criteria, DSA stands for deterministic sensitivity analysis and FAEP stands for Finnish agri-environmental programme

2.2. System dynamics

The optimisation problem of simultaneous N and P fertilisation is essentially a dynamic problem. This results from the fact that most of the P contained in crops originates from the P accumulated in soil as a plant available soil P reserves (Hooda et al., 2001; Sharpley, 1986; McLaughlin et al., 1988) It is a general observation that the crop yield is greater the higher is the STP class of the soil (see e.g., Analogides and Rendig, 1972; Barrow, 1980; Dodd and Mallarino, 2005). In addition, Iho and Laukkanen (2009) showed that the optimal annual P fertilisation rate changes in time response to changes in STP and it has been shown in multiple studies that the P fertilisation increases the yield on soil with low STP but on soil with high STP levels the crop response to applied P is not expected (see e.g., Engelstad and Terman, 1980; Saarela et al., 1995; Mallarino and Prater, 2007; Valkama et al., 2011).

We assumed that the P fertilisation has a direct effect on an annual yield as well as an indirect positive effect on STP due to the accumulation of P into the soil and an indirect negative effect on STP through the crop-uptake (the proportion of P that is removed from soil in yield). In contrast, N is assumed to have an indirect effect on STP through the crop-uptake. In addition, we assumed that whereas the P response is negatively associated with the current STP level of the soil, the N response is not associated with the STP. The phenomenon is illustrated with a simple stock and flow model represented in Figure 2, where STP is the accumulating stock (the state variable) and fertiliser inputs (the control variables) represent system inflows while the crop output represent the system outflow.

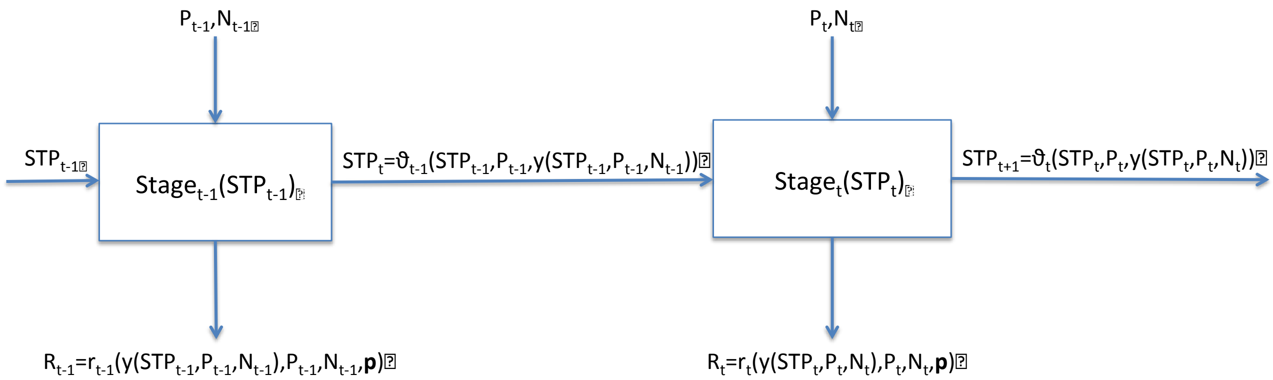


Fig. 2, Stock and flow model of the dynamic agricultural system (based on Figure presented in Hardaker et al., 2015, p.206). In the system each stage is characterised by a STP-level, which develops according to a transition function ϑ. The inflows consist of annual P and N inputs while the output consist of annual returns R_t, which are a function of profits from annual yield and input costs.

Fertilisation decisions will be made over T stages. The stages represent different time periods, years, in the *planning horizon* of the problem. Each stage is characterised by a current STP level (STP is the state of the stage). The current stage STP and the input inflows (N and P fertilisation) and the output outflows (barley yield) determine the level of STP which is transferred to the next stage via the transition function, which is defined with the following difference equation: $STP_{t+1} = \vartheta\left(SPT_t, P_t, y(STP_t, P_t, N_t)\right)$, where ϑ is a transition function. In addition to the current STP level, each stage is characterised by a current crop yield, which is a function of the current STP level and the fertilisation decisions. Consequently, each stage produces a stage return, which is a function of the current crop yield, the decision variables and the prices: $R_t = r_t(y(STP_t, P_t, N_t), P_t, N_t, \boldsymbol{p})$ where r_t is a stage return function (which is defined below in a section 2.7.) and $\boldsymbol{p}$ is a price vector.

Hence, the dynamic system model is a combination of two models: a yield response model and a state model, i.e. the transition function (Wallach et al., 2006). When economic components are added to the yield response model, the coupled system of models can be utilised in dynamic economic optimisation, where P and N fertilisation acts as control variables and STP acts as a state variable. The applied models are specified in following sections.

2.3. Specification of the yield response model

The yield response model consists of three multiplicative elements: the first element defines the P-control yield (kg ha^{-1}) (denoted by Y_{P0}) as a function of STP (mg l^{-1}). The P-control yield is the yield without added P fertiliser, but fertilised with NK; the second element defines the yield response (%) to P fertiliser as a scaling factor (denoted by ω_P) that scales the first element up depending on the applied P rate (kg P ha^{-1}) and the STP level; the third element defines the yield response (%) to N fertiliser (kg N ha^{-1}) as a scaling factor (denoted by ω_N) that scales the previous elements up or down depending on whether the applied N rate is lower or higher than the average N rate in P-experiments. The element is decreasing function of N-control yield (kg ha^{-1}), i.e. the yield without added N, but fertilized with PK (denoted by Y_{N0}), because Valkama et al. (2013) noticed a significant negative association between the yield response to N fertiliser and N-control yield. The following nonlinear regression model gives the complete specification of the model on the relationship between the yield and the predictors:

$$Y_i = Y_{P0}\left(\theta_{1,j}, STP_i\right)\omega_P\left(\theta_{2,j}, P_i, STP_i\right)\omega_N\left(\theta_{3,j}, N_i, Y_{N0,i}\right) + \varepsilon_i, \qquad (1)$$

where Y_i is the observed yield in an experiment i with $i = 1, \dots, k$, and $\theta_{l,j}$ indicates the parameter with the subscript $l \in [1,3]$ denoting a model element, and subscript j denoting a parameter number and ε_i indicates model residual error.

Eq.1 is assumed to be finite, nonnegative, real valued, and single valued for all possible combinations of inputs (A1), everywhere continuous, strictly increasing, strictly quasiconcave and everywhere twice continuously differentiable (A2), and subject to the ‘law of diminishing marginal productivity’ (A3). It was further assumed that the first model element in [1] is a concave function of STP (A4), the second model element in [1] is a concave function of P and decreasing function of STP (A5), the third model elements is concave function of N and a decreasing function of N-control yield (A6), the yield and the response to any variable must converge to maximum plateau with zero marginal product as input utilisation increases (A7), and the model residual errors are independently and identically distributed according to a normal distribution with a zero mean and constant variance (A8). This assumption implies further that we assumed the yields to be normally distributed also, although the normality of the yield distribution is not accepted unanimously (e.g. Just and Weninger, 1999).

The assumption (A5) is motivated by the fact that a plant takes all the P from soil solution where it is in a plant available form. When P fertiliser is added to the soil, it also moves into the soil solution where plant uses it to grow (Busman et al., 2002). As the growth follows the von Liebig's law of the minimum, more there is P in the soil solution, i.e. higher is the STP, less limiting factor P is and less growth can be expected to be gained from applying additional amount of P fertiliser.

The assumption (A6) is hypothesised to be reflecting the effect of N soil stock; if the yield without added N is high, there might be a sufficient amount of plant-available N in the soil. According to the law of the minimum, the increase in yield for N fertilisation is the less the more there is plant-available N in the soil. In practical applications, i.e. in predictive simulations and economic optimisation, Y_{N0} was treated as a parameter rather than as a variable, because we did not have a transition function for describing the N dynamics in the soil. We hypothesised that the N-control yield must be lower than the P-control because in general N has stronger effect on yields than P or STP (e.g., Carlgren and Mattson, 2001; Salo et al., 2013). Therefore it was fixed to a level of 2/3 of the P-control yield.

The assumption (A7) is based on general agronomic principle stating that the yield cannot grow boundlessly because there are fixed biological or agronomic (or both) limitations for the crop growth (see e.g., Niklas, 2006). This assumption is particularly important for the first model element in [1], because the model element determines the yield as a function of STP, which is state variable in dynamic optimisation. However, in the case of N fertilisation we consider two kinds of cases: (1) the yield converges a maximum level with a diminishing rate and (2) the yield reaches its maximum at a certain N level after which it starts to decline. The latter case is based on the fact that various crops show a negative response to N at high levels (Jensen and Schjoerring, 2011). Such a phenomenon is not observed relating to P fertilisation or STP levels (Saarela et al., 1995).

2.4. Specification of the transition model

We obtained the explicit form of the transition function from Uusitalo et al. (2016):

$$\Delta STP_t = \delta_1 + \delta_2 P_{bal,t} + \delta_3 P_{bal,t} STP_t - \delta_4 STP_t, \qquad (2)$$

where $\Delta STP_t = STP_{t+1} - STP_t$ and δ_q with $q \in [1,4]$ are parameters. The specification was applied because Uusitalo et al. (2016) estimated different functions for different soil textures. Since the functions of Uusitalo et al. (2016) have P-balance as one of the model arguments, we utilised a function from Iho and Laukkanen (2012) for the P-balance description:

$$P_{bal,t} = P_t - (\beta_1 \log(STP_t) + \beta_2) Y_t, \qquad (3)$$

where β_1 and β_2 are parameters and the term $\beta_1 \log(STP_t) + \beta_2$ determines the phosphorus concentration of the crop yield (Iho and Laukkanen, 2012). Thus, the annual P-balance is a difference between annual P input and P that is removed via the crop-uptake. When we combine these functions, i.e. insert a function [3] into [2], we get the following specification for the transition model:

$$\Delta STP_t = \delta_1 + \delta_2(P_t - (\beta_1 \log(STP_t) + \beta_2)Y_t) + \delta_3(P_t - (\beta_1 \log(STP_t) + \beta_2)Y_t)STP_t - \delta_4 STP_t \qquad (4)$$

The properties of the transition function are examined further in Appendix 5. When the models [1] and [4] were combined we got the dynamic agricultural system model described in Figure 2. However, before the system model could be utilised in numerical applications, the explicit forms for the yield response models were to be estimated from the empirical data.

2.5. Fertiliser field experiment data

In order to estimate the yield response models to be utilised in the dynamic system model, six separate datasets were applied: one for each model element in [1] for both soil textures. The applied datasets consisted of the results of the Finnish fertiliser field experiments of cereals, which were analysed via a meta-analysis method. Valkama et al. (2011, 2013) provide detailed description of these datasets. To model the combined impacts of N and P fertilisation on crop yield, we standardised the data in such a way that the observations obtained from the two distinct sets of empirical experiments with different control yields were comparable (see Appendix 2). In addition, for the estimation of the first model element in [1], following Myyrä et al. (2007), we utilised the data that was provided in the report by Saarela et al. (1995) because there was limited information regarding the relationship between STP level and P-control yield in the dataset obtained from Valkama et al. (2011). The descriptive statistics of the applied datasets are provided in Tables 1-3 in the Appendix 1.

We also gathered historical field experiment data for evaluating the prediction capability and generalisability of the estimated models. The pooled (mixture of cross-sectional and time series) datasets consisted of unpublished reports of compound NPK fertiliser field experiments conducted at MTT (Agrifood Research Finland) and its research stations. The datasets consisted of 28 short- and long-term (mostly short-term) experiments conducted in Finland between 1964 and 1988 at 10 sites on clay and coarse soils. Most of the experiments were conducted on clay soils. It must be emphasized that it was particularly important to validate the model with datasets consisting of NPK field experiment data. By applying such data for validation we could evaluate how the model, consisting of separate elements estimated for P, N and STP response, predicted a situation where all of these variables were changing simultaneously. Therefore, the goodness-of-fit to the validation dataset could be considered as more crucial evaluation of the model performance than the goodness-of-fit to the initial datasets. The descriptive statistics of the gathered validation datasets are provided in Table 4 in the Appendix 1.

All the applied datasets are provided in the Supplementary material.

2.6. Estimation, ranking and evaluation of the models

We utilised weighted nonlinear least squares method for estimating the models. We weighted the observations with the duration of the respective experiment, since the reported yields were averaged over the experiment. Therefore, longer experiments produced more reliable results, since these were less affected by random variation (Esala and Larpes, 1984). Since there are typically multiple models that can be fitted successfully into dataset (Cerrato and Blackmere, 1990), we examined several functional forms (presented in the article by Griffin et al. (1987)). As a result of this process we obtained the set of candidate models. The selected functional forms were altered or combined (or both) in various ways. We choose three to five models for each model element in [1] and both soil textures for further examination out of the numerous iteratively obtained alternatives.

We ranked the estimated models via the modified second order variant of Akaike's (1973) information criteria (denoted by AIC_c), which is bias-corrected for small samples. In addition, the *relative likelihoods* were utilised to determine how much better the best model is compared to other models. We determined the structural uncertainty to be considerable in the case where the relative likelihood for a best model was < 1.5. We also utilised traditional frequentist statistical goodness-of-fit analysis because AIC is unable to indicate how well the models fit the dataset used for estimation (Burnham and Anderson, 2002). It must be noted that to measure the model predictive accuracy, we utilised Pearson's product-moment correlation coefficient (r) and the adjusted squared correlation coefficient (r^2_{adj}), although the utilisation of r^2 as a measure of predictive accuracy for nonlinear models is often criticised since it cannot be interpreted in the same way as in the case of linear models (e.g., Spiess and Neumeyer, 2010). We provided these measures as an approximation of the predictive accuracy, since the measures are frequently utilised, which enables the communication of the model performance (Bennett et al., 2013).

2.7. Dynamic economic optimisation problem

We utilised dynamic programming method for determining the optimal fertilisation decisions over the planning horizon. The annual profit was defined with the following typical stage return function $r_t = p_y y(STP_t, P_t, N_t) - p_P P_t - p_N N_t$ where y is the estimated yield response function and p_y, p_P and p_N are the exogenous prices for the crop yield, P and N fertilisers, respectively. Returns from each stage must be discounted in order to obtain their present value. We defined the discount factor as follows: $\beta = \frac{1}{1+\rho}$, where ρ is a discount rate. The objective was to maximise the net present value of the discounted sum of the annual returns over the planning horizon. We considered that the planning horizon consisted of two phases: the production period and the post-production period. The final stage of the production period produces the scrap value (or bequest value) that was assumed to generate profits infinitely: $S_T = \sum_{t=0}^{\infty} \beta^{T+t}\, r_{T+t}$ (e.g., Sydsaeter, 2005; Ikefuji et al., 2010). However, the value of the S_T obviously needs to be approximated for practical reasons. We utilised the following approximation for the sum of the discounted annual values of the scrap value function: $S_T = \beta^T \sum_{t=0}^{\infty} \beta^t\, \pi_{T+t} \approx \beta^T \frac{\pi_T}{\rho}$, which originates from the continuous discounting over infinite horizon since $\int_{t=0}^{\infty} e^{-\rho(T+t)} \pi_T dt = e^{-\rho T} \frac{\pi_T}{\rho}$. We formalised the optimisation problem as a recursive finite horizon dynamic programming problem, which we solved numerically by iterating the Bellman's equation (Bellman, 1957):

$$J(STP_t) = \max_{P_t, N_t} \sum_{t=1}^{T-1} \beta^t\, r_t + S_T$$

$$s.t.\, STP_{t+1} \leq \vartheta\big(STP_t, P_t, y(STP_t, P_t, N_t)\big), STP_1 \text{ given}, P_t, N_t, SPT_t \geq 0, \quad (5)$$

where $J(STP_t)$ is called a value function, which determines the present value of net returns from carry-over STP_t obtained following the optimal control trajectory $\{P_t^*, N_t^*\}_t^{T-1}$. The value function does not depend on the control variables because when decision maker is behaving optimally, the control variables are a function of the state variable (Wälde, 2011).

2.8. Uncertainty and sensitivity analysis

Lastly, we applied principles of multimodel inference (Burnham and Anderson, 2002) in order to evaluate the effect of model structural uncertainty on economic optimisation analysis and to examine the robustness of the comparable models. Such a model inter-comparison is often suggested in previous literature (Hildreth, 1955; Griffin et al., 1987; Cerrato and Blackmer, 1990; Asseng et al., 2013). In addition, the parameter uncertainty was represented via deterministic sensitivity analysis (DSA), in which parameter values were varied manually to their 95% confidence bounds, in order to test the sensitivity of the optimisation results for all the parameters (Briggs et al., 2012). Nevertheless, since we were also interested in the distribution of realised output values when the input values were at their optimal levels, a Monte Carlo analysis was carried in addition to DSA analysis. To carry out a Monte Carlo analysis, we assumed that the probability distribution of all the model parameters followed a normal distribution. In addition to uncertainty analysis, we carried out a sensitivity analysis in order to determine how sensitive the optimisation results were with respect to economic parameters, the initial STP level and the N-control yields, since those parameters were clearly subject to uncertainty.

3 Results

3.1. Estimated yield response models

The estimated model candidates for each model element in [1] are presented in Appendix 3. The best models for the first model element in [1] were power/Spillman and power/square root-plus-plateau models for coarse and clay soils, respectively. The best model for the second model element in [1] was square root/quadratic and square root/logistic for coarse and clay soils, respectively. In the case of coarse soils, the best model for the third model element in [1] was logarithm/quadratic. For clay soils the best model for the third model element in [1] was square root/quadratic (see Appendix 3). Table 1 shows that all the estimated model parameters were significant at 5% level. The least significant parameters were the two parameters in the third model element for clay soils as well as one parameter in the second model element for coarse soils.

Table 1
Estimated parameters and their summary statistics for the best models for the respective models elements in [1]

Model (model element), Soil texture	Parameter	Estimate	SD	t -value	P (>$\|t\|$)
Power/Spillman (1), Coarse soils					
	$\theta_{1,1}$	3883	360	10.79	5.05e-10
	$\theta_{1,2}$	0.80	0.050	15.87	3.63e-13
Square root/Quadratic (2), Coarse soils					
	$\theta_{2,1}$	0.0427	0.0064	6.71	5.98e-9
	$\theta_{2,2}$	0.0106	0.0044	2.42	0.0184
Logarithm/Quadratic (3), Coarse soils					
	$\theta_{3,1}$	0.649	0.047	13.81	< 2e-16
	$\theta_{3,2}$	0.02402	0.0026	9.28	3.35e-13
	$\theta_{3,3}$	3.653e-08	4.750e-09	7.69	1.66e-10
Power/square root/Plateau (1), Clay soils					
	$\theta_{1,1}$	3233	251.7	12.81	7.97e-10
	$\theta_{1,2}$	0.5903	0.03576	16.51	1.80e-11
Square root/Logistic (2), Clay soils					
	$\theta_{2,1}$	0.0375	0.00758	4.943	1.5e-05
	$\theta_{2,2}$	0.0999	0.03663	2.727	0.0095
	$\theta_{2,3}$	0.0759	0.02093	3.629	0.0008
Square root/Quadratic (3), Clay soils					
	$\theta_{3,1}$	0.197	3.657e-02	5.399	5.68e-06
	$\theta_{3,2}$	-0.005299	2.229e-03	-2.377	0.0234
	$\theta_{3,3}$	9.322e-08	4.273e-08	2.182	0.0364

The predictive accuracy and efficiency was best for the third model element in [1] for coarse soils, whereas for clay soils the most accurate and efficient model element was the second model element (Table 2). In contrast, the statistics measuring the accuracy of model prediction clearly indicated worst fit for the first model element for both soil textures. The fit was particularly poor for the clay soils; the correlation for the model predictions and the observations was non-significant at 5% level, although the correlation between the STP and the P-control yield was significant in the initial dataset (p-value=0.040). The most biased model was clearly the first model element for coarse soils (Table 2). In the case of clay soils models, the most biased model element was the third model element. For both soil textures, the high bias was related to the relatively high variation within the respective datasets: for coarse soils the CV (SD/average) for the STP response dataset was 37% and for clay soils the CV for the N response dataset was 19.7% (see Appendix 1). The second model element for both soil textures was the least biased model elements, reflecting the relatively minor variance within the respective datasets; the CV for coarse soils was 10.8% and for clay soils it was 5.8% (see Appendix 1).

Table 2

The best models for the respective models elements in [1] and the associated goodness-of-fit statistics*

Soil Texture	Model (model element)	r	P (>$\|t\|$)	r^2	r^2_{adj}	IA	RMSE	MB	MAE
Coarse	Power/Spillman (1)	0.61	0.0021	0.37	0.34	0.69	941 (29%)	90 (2.8%)	789 (24.5%)
Coarse	Square root/quadratic (2)	0.79	6.056e-15	0.62	0.61	0.86	0.073 (6.7%)	-0.011 (1.1%)	0.053 (4.9%)
Coarse	Logarithm/quadratic (3)	0.87	< 2.2e-16	0.76	0.75	0.92	0.078 (9.0%)	-0.007 (0.8%)	0.071 (8.2%)
Clay	Power/square root-plus-plateau (1)	0.42	0.0838	0.18	0.12	0.55	646 (17%)	-30.4 (0.8%)	474 (13%)
Clay	Square root/logistic (2)	0.75	1.4e-08	0.56	0.55	0.85	0.041 (3.9%)	0.0024 (0.2%)	0.032 (3%)
Clay	Square root/quadratic (3)	0.68	4.251e-06	0.47	0.45	0.80	0.157 (17%)	-0.040 (4%)	0.130 (14%)

* *r* is Pearson's product-moment correlation coefficient measuring the correlation between the observations and the model predictions, r^2 is a squared correlation coefficient, r^2_{adj} is a adjusted coefficient of determination, *IA* is a index of agreement, *RMSE* is root mean squared error, *MB* is a mean bias and *MAE* is a mean absolute error. The p-value refers to a correlation test. The statistics as a percentage of the observed mean values are found in brackets

3.2. Predictive yield surfaces of the integrated models and the interactions with the transition model

After the best models for the sub-model components were determined, we were in a position to present the integrated models where individual model elements were multiplied with each other. For coarse soils the explicit form of the integrated model is presented in Eq.6.

$$\bar{Y}_{coarse} = \theta_{1,1} STP^{\frac{1}{\theta_{1,1}}}\left(1-{\theta_{1,2}}^{STP}\right)\left(\frac{\theta_{2,1}\sqrt{P}}{1+\theta_{2,2}STP^2}+1\right)\frac{\theta_{3,1}+\theta_{3,2}(log(N+1))^2}{1+\theta_{3,2}{Y_{N0}}^2}, \qquad (6)$$

where the first subscript of the parameter denotes the model element and the second subscript denotes the parameter number. For clay soils the explicit form of the integrated model is presented in Eq.7.

$$\bar{Y}_{clay} = \theta_1\left(STP^{\theta_2}+1\right)\left(1+\sqrt{STP}\right)^{-1}\left(\frac{\theta_{2,1}\sqrt{P}+\theta_{2,2}}{1+e^{\theta_{2,3}STP}}+\omega_{P,min}\right)\frac{\theta_{3,1}\sqrt{N}+\theta_{3,2}}{\theta_{3,3}{Y_{N0}}^2+1}. \text{ when } STP < STP^*$$

$$\bar{Y}_{clay} = max\bar{y}_{P0}\left(\frac{\theta_{2,1}\sqrt{P}+\theta_{2,2}}{1+e^{\theta_{2,3}STP}} + \omega_{P,min}\right)\frac{\theta_{3,1}\sqrt{N}+\theta_{3,2}}{\theta_{3,3}{Y_{N0}}^2+1}, \text{ when } STP \geq STP^* \quad (7)$$

We examined the predictive performance of the models graphically and analytically. The most critical aspect of the integrated models was the interaction of the first and second model element because the first is an increasing function and a second is a decreasing function of STP; since the derivative of the integrated model with respect to STP is the following: $\frac{\partial \bar{Y}}{\partial STP} = \left(\frac{\partial y_{P0}}{\partial STP}\omega_P + y_{P0}\frac{\partial \omega_P}{\partial STP}\right)\omega_N$ and $\frac{\partial y_{P0}}{\partial STP} > 0$ while $\frac{\partial \omega_P}{\partial STP} < 0$, we have a following condition: $\frac{\partial \bar{Y}}{\partial STP} > 0$ if $\frac{\partial y_{P0}}{\partial STP}\omega_P > y_{P0}\frac{\partial \omega_P}{\partial STP}$ and $\frac{\partial \bar{Y}}{\partial STP} < 0$ if $\frac{\partial y_{P0}}{\partial STP}\omega_P < y_{P0}\frac{\partial \omega_P}{\partial STP}$. This implies that while the yield without added P (i.e. P control yield) must be higher for high STP levels compared to low STP levels, there can be conditions under which the P fertilisation raises the yield higher for low STP levels than what is the maximum yield for higher STP levels. The yield increasing effect of P fertilisation vanishes for higher STP levels and the yield converges the maximum plateau from above. The Figure 3 illustrates that this phenomenon takes place for coarse soils. The Figure 3 also illustrates the effect of N fertilisation; since N fertilisation is not a function of STP and there is no interaction between N and P fertilisation in this model formulation, the N fertilisation raises the yield curve but it does not affect the shape of the curve. In the case of clay soils the yield is a monotonic function of STP on a whole domain of possible STP levels for all P rates because the P response is low (even for low STP levels) compared to that observed on coarse soils (see Appendix 2, Fig.1c, d).

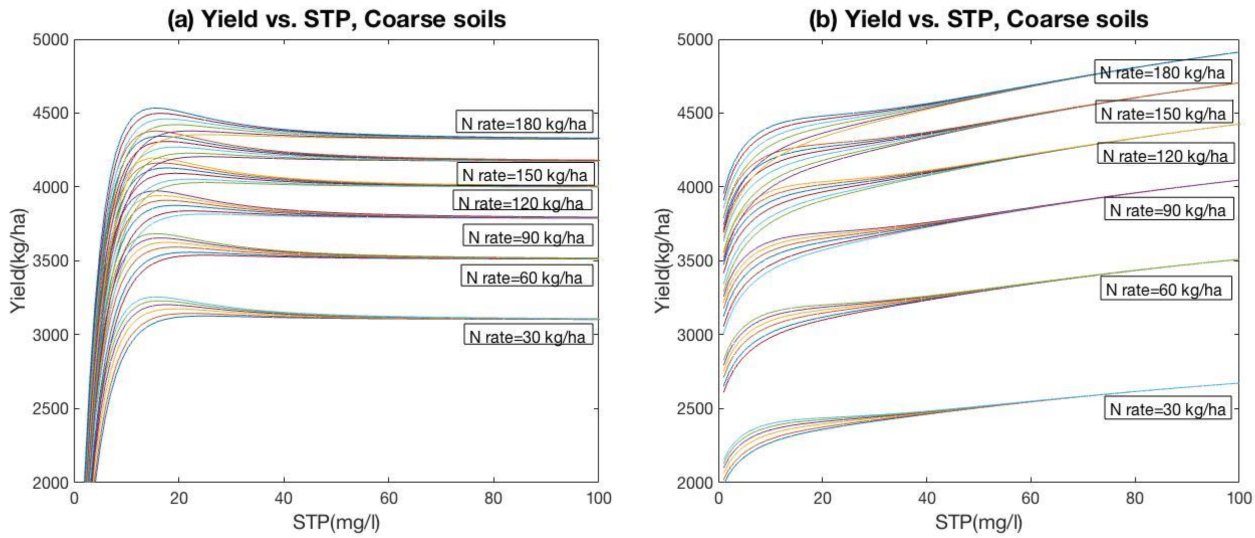
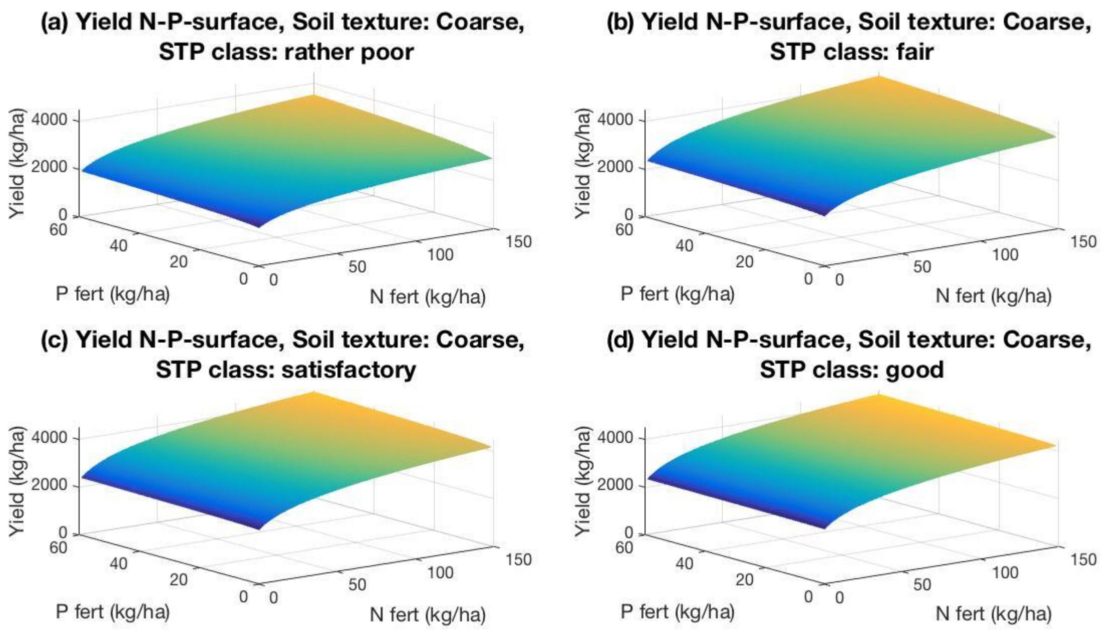


Fig. 3. The yield as a function of STP for various P and N rates. The P rates are 5, 10, 20, 30, 40, 50 and 60 kg/ha.

Figure 4 represent the simulated yield N-P-surfaces for various STP levels (level curve version of the Figure 3 is represent in Appendix 6 (Isoquants)). For rather poor STP class soils the coarse soils model gave lower yield surfaces than the clay soils model, which resulted from the applied functional forms for the first model element in [1], as the P-control yield was higher for lower STP levels in the case of clay soils (see Appendix 3, Fig. 1a, b). The relative effect of P was more crucial for coarse soils compared to that for clay soils, whereas the relative effect of N was more crucial for clay soils compared to that for coarse soils (Fig. 4). From Figure 4 one may also notice the absence of the interaction effects of the N and P fertilisers. These effects could not be captured, since the effects of the fertilisers were estimated from the separate datasets of field experiments where interactions were omitted. Nevertheless, when the models are coupled with the transition functions, the interactions enter the models, since both fertilisers affect the development of the STP through the crop-uptake in the following way: the crop-uptake is higher for higher N rates, STP is lower for higher crop-uptake, and P response is higher for lower STP. Within this modelling framework, however, the P does not affect the N response since the N response is not a function of STP. The analytical properties of the models specified in [6] and [7] are examined further in Appendix 6.



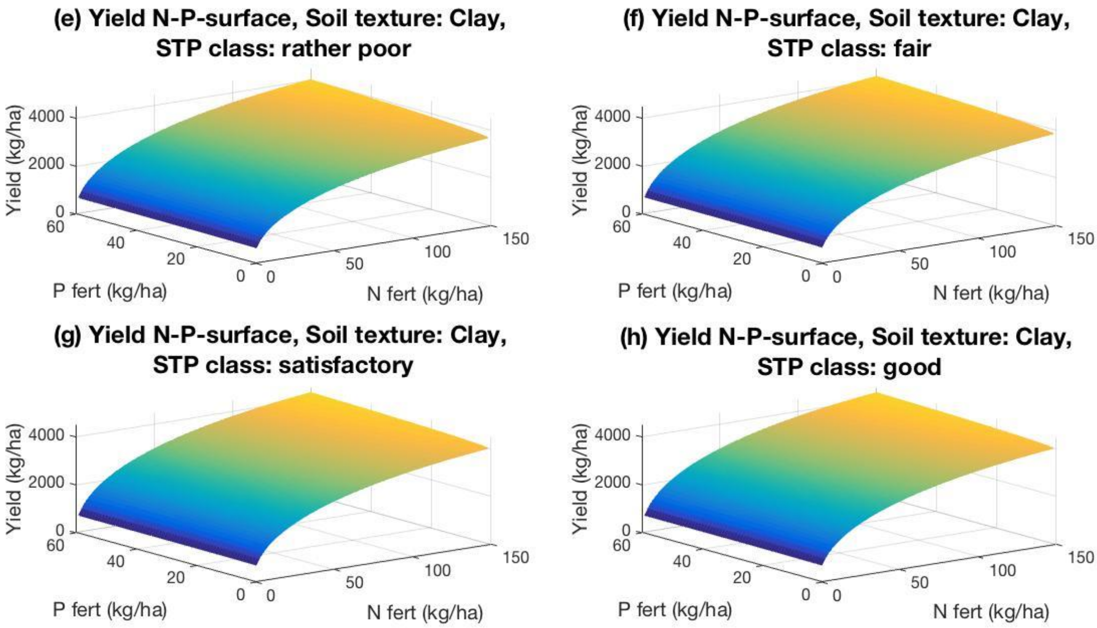


Fig. 4. Simulated yield N-P-surfaces for both soil textures for various STP levels (STP level rather poor is 3 mg l^{-1} for clay and 5 mg l^{-1} for coarse soils, STP level fair is 6 mg l^{-1} for clay and 10 mg l^{-1} for coarse soils, STP level satisfactory is 11.5 mg l^{-1} for clay and 17.5 mg l^{-1} for coarse soils and STP level good is 20 mg l^{-1} for clay and 28.5 mg l^{-1} for coarse soils)

3.3. Model validation results

We evaluated the integrated models by comparing the simulated predictions to observations obtained from the NPK fertilisation field experiments, where both N and P were given simultaneously in different dozes on soil characterised by various STP levels. In this comparison, observed N and P rates as well as the STP levels were applied as input vectors for the models. For both models the distribution of the model error residuals was not significantly different from a normal distribution: p-value=0.1129 for coarse soils and p-value=0.371 for clay soils (Shapiro-Wilk normality test). In addition, the prediction accuracy was high, particularly in the case of coarse soils (Table 3). The model efficiency was equal and high for both models. Among the statistics measuring model biasness, RMSE and MAE indicated lower bias for coarse soils model, whereas the MB was lower for clay soils model (Table 3).

Table 3
Validation statistics for the best models*

Model	Soil texture	r	P (>$\|t\|$)	r^2	r^2_{adj}	IA	RMSE	MAE	MB
Power/Spillman –square root-square root	Coarse	0.79	1.201e-08	0.63	0.63	0.85	537 (15 %)	458 (12 %)	263 (7.2 %)
Power/square root-plus-plateau -square root/logistic-square root/quadratic	Clay	0.74	2.969e-13	0.55	0.55	0.85	888 (27 %)	666 (20 %)	140 (4.3 %)

*The p-value refers to a correlation test. In brackets are the statistics as a percentage of the observed mean values

The bias of the residuals in the case of coarse soils model appears in Figure 6, as most of the observations were higher than the predictions, particularly those with numbers 20-34 (Fig. 6b). These high observations came from one experiment, where the annual yields were high most likely due to favourable agri-environmental conditions. These observations influenced the model performance since the validation dataset for coarse soils had fewer observations (n=35) than the dataset for clay soils (n=68), as was mentioned above (see section 2.5 and Table 4 in Appendix 1). Nevertheless, the model appeared to predict the observations patterns relatively accurately (Fig. 6b) and the residuals were not significantly different from a normal distribution (Fig. 6c), as was also suggested by the Shapiro-Wilk normality test.

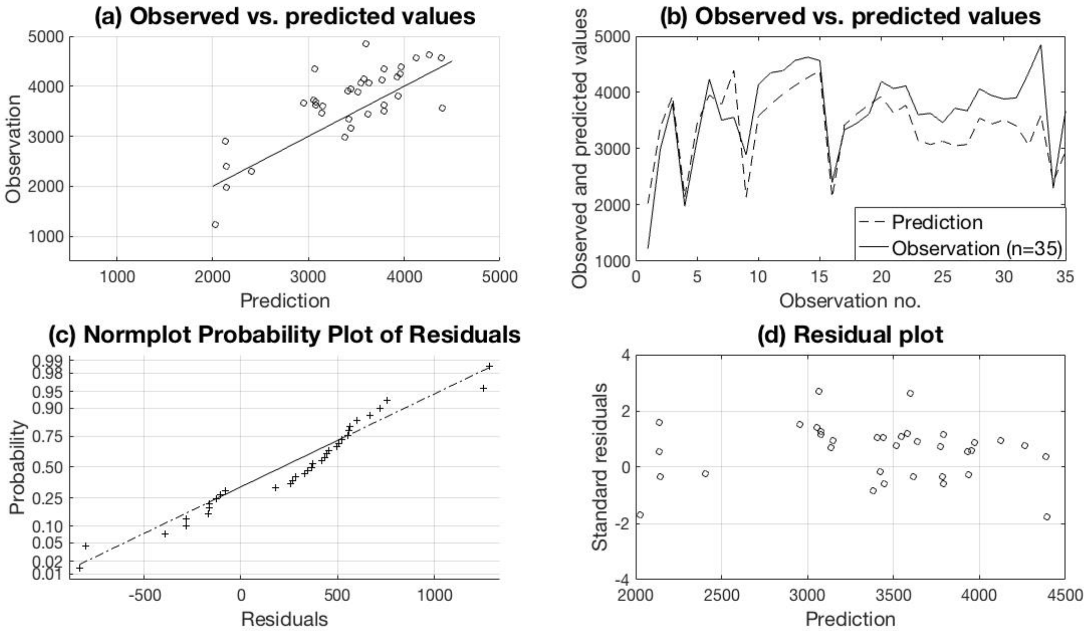


Fig. 6. Graphical model validation results for best coarse soils model: (a) plots how the observations are scattered around the 1:1 line, (b) plots how the model predictions produce the observation patterns, (c) plots the normality of the residuals and (d) plots the standardised residuals against the model predictions.

The accuracy of model prediction and the distribution of the residuals between observations and predictions for the clay soils model are illustrated in Figure 7. The model poorly predicted the highest and lowest yield values (Fig. 7b), since the model operated with the expected average yields, the extreme values were omitted by the definition and the variation within the validation dataset regarding clay soils was relatively high compared to that observed within the coarse soils (e.g., for yields the CV-ratio was 32% for clay soils whereas the CV ratio was 21.4% for coarse soils). Figure 5c shows a normal probability plot and verifies the result obtained via the Shapiro-Wilk normality test. It should be noticed, that there was a cluster of observations that were underestimated by the model with a systematic error, which was the primary reason for bias within the predictions (Fig. 7a, d). These observations consisted of control yields as well as other yields associated with low input rates.

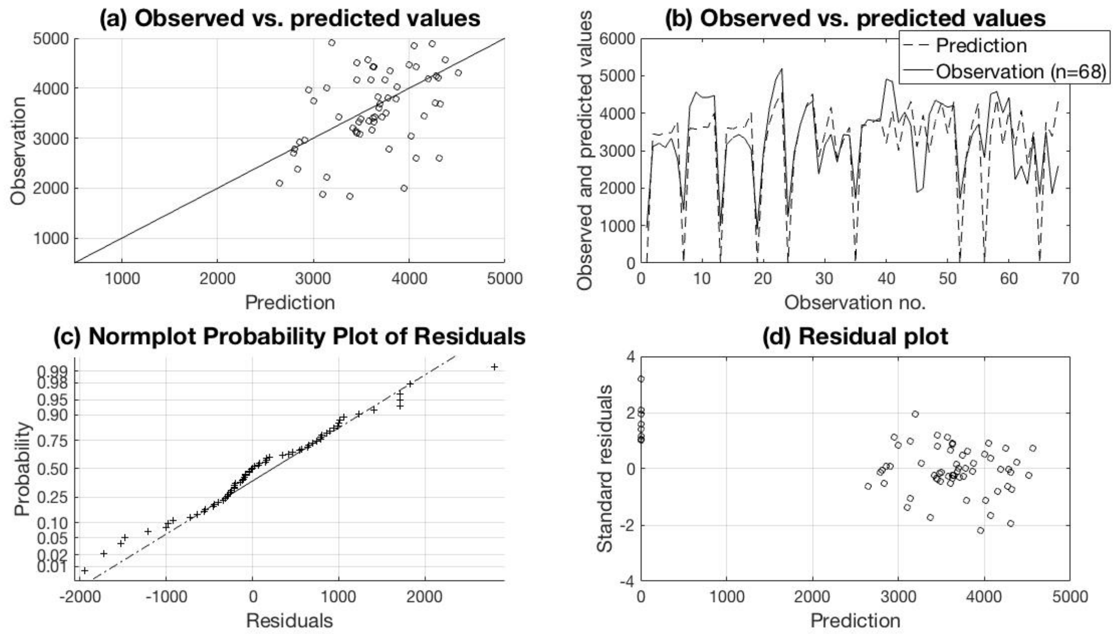


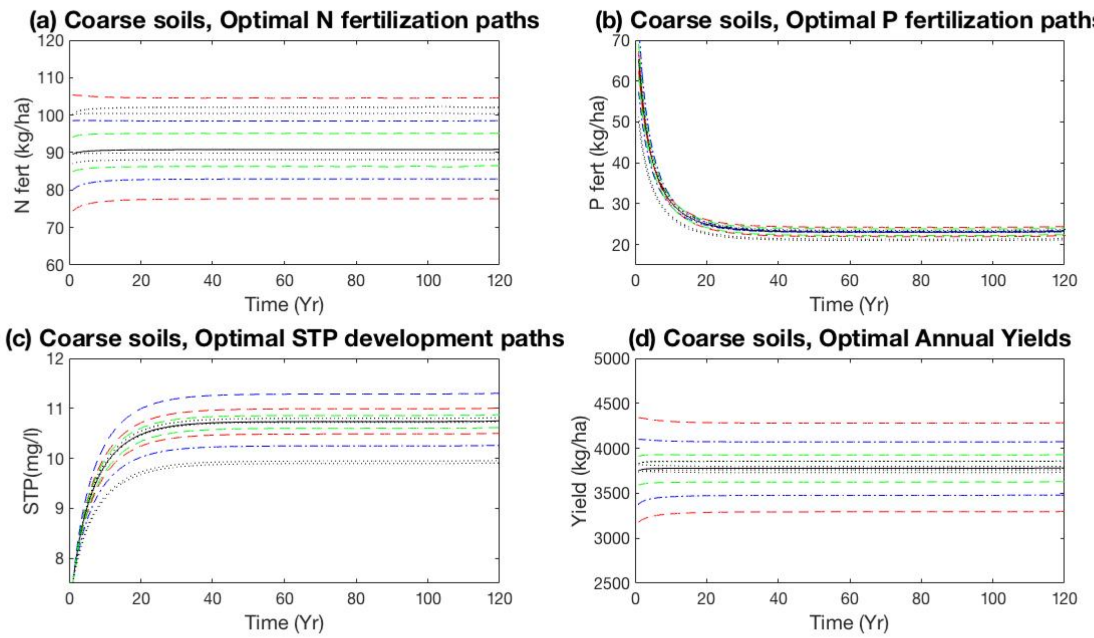
Fig. 7. Graphical model validation results for best clay soils model.

3.4. Results of economic optimisation, the structural uncertainty analysis and the DSA

Finally, when the economic factors were added to the system of the yield response models and the transition models, we were in a position to perform the bio-economic analysis. We represent the results of the dynamic economic optimisation, the structural uncertainty analysis and the DSA in the same section because the results were obtained simultaneously. The results form the Monte Carlo analysis and the sensitivity analysis are provided in the Appendix 7 and 8, respectively.

The optimal paths for each variable generated by the set of candidate models as well as the 95% confidence intervals for the best models are illustrated in Figure 8 for both soil textures. The optimal paths converged to the steady state within the time horizon in all the cases suggesting that the models were suitable for dynamic economic optimisation. The optimal N rate was 45% higher for clay soil than for coarse soils. Instead, the optimal P rate was 182% higher for coarse soils than for clay soils. As a direct consequence, the optimal STP level was 161% higher for coarse soils than for clay soils. The optimal steady state STP class for both soil textures was fair (range of the class is 7-13 mg l^{-1} for coarse soils and 4-8 mg l^{-1} for clay soils (Ylivainio et al., 2014)). The high P rate for coarse soils in the beginning of the planning horizon stemmed from the fact that the initial STP level was further away from the steady state level in the case of coarse soils compared to that in the case of clay soils. The optimal annual yield was almost the same for both soil textures; the yield was 2.1% higher for clay soils than for coarse soils, whereas the optimal yield given by the best model was 2.4% higher for coarse soils than for clay soils (Fig. 8d, h).

The parameter uncertainty had a greater effect on economic optimums than the structural uncertainty for both soil textures (Fig. 8). In addition, the parameter uncertainty had a greater effect on optimums for clay soils than for coarse soils (Fig. 8). For coarse soils, most of the parameter uncertainty resulted from the first model element, which was the most biased model among the coarse soils models, whereas for clay soils most of the parameter uncertainty stemmed from the third model element, which was the most biased model among the clay soils models (see section 3.1, Table 1). The structural uncertainty also had a greater effect on optimums in the case of clay soils compared to coarse soils, which is particularly observable from the CV-ratios presented in Table 4. In the case of the clay soils models, the structural uncertainty originated from the choice of the third model element; when the applied functional form was power/quadratic instead of square root/quadratic, the economic optimums were higher, particularly optimal N rates (Fig. 8e). The choice of first and second model elements had only a minor effect on economic optimums. For coarse soils the structural uncertainty also stemmed from the choice of a third model element. Whether the applied functional forms would have been square root/quadratic or power/quadratic instead of logarithm/quadratic, the optimal N-rates would have been higher and P rates and STP levels lower (Fig. 8a-c). For optimal P rates and STP levels, the effect of structural uncertainty was greater than the effect of parameter uncertainty (Fig. 8b, c). Instead, the structural uncertainty had only a minor effect on optimal yields relative to the effect of the parameter uncertainty (Fig. 8d).



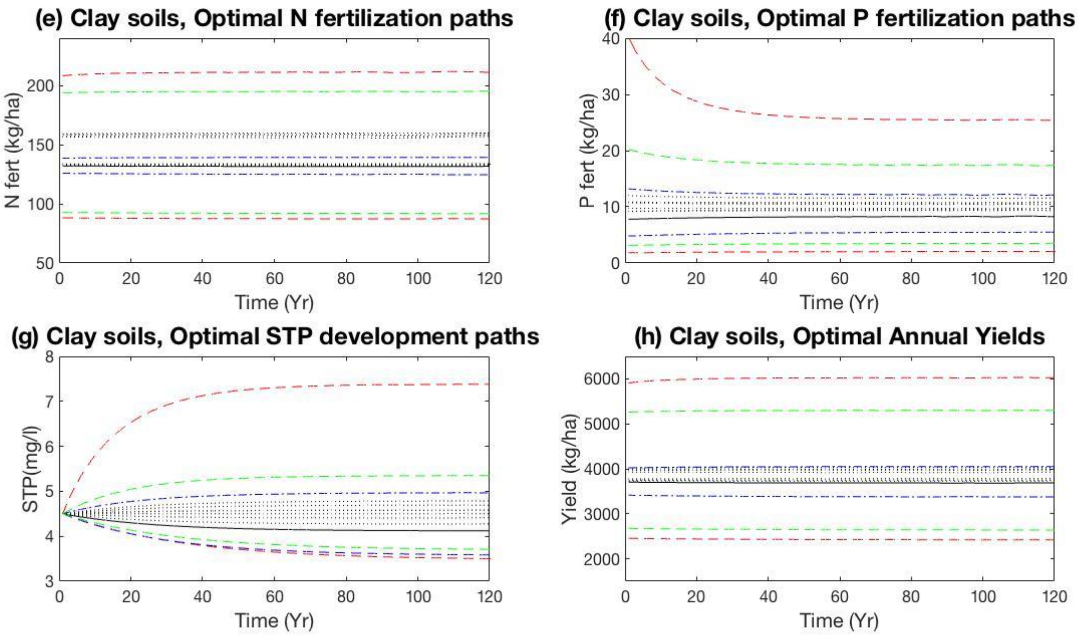


Fig. 8. Optimal N and P fertilisation paths, STP development paths and annual yields given by nine models on coarse soils (a-d) and eight models on clay soils (e-h). 95% confidence bounds are shown for the best model when all the model parameters are changed (red dashed lines), the model parameters of the first model element are changed (blue dashed lines) and the model parameters of the third model element are changed (green dashed lines). The solid line is the best model and the other models are presented as dotted lines. The fixed parameters are ρ =discount rate (3.5%), N price=N fertilisation price (0.91 €/kg), P price=P fertilisation price (1.99 €/kg), Y price=price for spring barley yield (0.12 €/kg) and STP_1=the initial STP-level of the soil (7.5 mg/l for coarse soils and 4.5 mg/l for clay soils models). Note that the y-axis does not begin from 0 in any of the figures (except in Fig.6f).

We also compared the obtained optimisation results to the maximum permissible N and P rates denoted by the Finnish agri-environmental programme, FAEP (Ministry of Agriculture and Forestry, 2014). The comparison showed that the obtained optimal N rates were on average (between the soil textures, 115.4 kg N ha^{-1}) 15.4% higher than maximum permissible N rates denoted by FAEP, whereas the optimal N rates were approximately 32% higher for clay soils and about 9% lower for coarse soils (Table 4). Compared to the maximum permissible P rates, the obtained economically optimal P rates for corresponding optimal STP levels were 44% higher for coarse soils but were 49% lower for clay soils. On average (between soil textures) the optimal P rates were 2.8% lower the maximum permissible P rates denoted by FAEP.

Table 4
The comparison of the optimums and the maximum permissible fertiliser rates denoted by FAEP*

Variable	Best model	Average	SD	CV	FAEP
		Coarse soils, power/Spillman-square root/quadratic –logarithm/quadratic:			
N rate (kg N ha^{-1})	90.8 (77.7, 104.6)	93.3	6.37	6.8 %	100
P rate kg P ha^{-1})	23.1 (22.0, 24.3)	22.5	1.08	4.8 %	16
STP (mg P l^{-1})/class	10.7 (10.5,11.0)	10.5	0.42	4.0 %	Fair
Yield (kg ha^{-1})	3780 (3290, 4280)	3780	50.1	1.3 %	4000

Clay soils, Power/square root-plus-plateau - square root/logistic-square root/quadratic:					
N rate (kg N ha^{-1})	132 (87.2, 211)	146	13.8	9.5 %	100
P rate (kg P ha^{-1})	8.2 (2.1, 25.4)	10.1	1.04	10 %	16
STP (mg P l^{-1})/class	4.1 (3.5, 7.4)	4.5	0.22	4.9 %	Fair
Yield (kg ha^{-1})	3690 (2420, 6020)	3860	137	3.5 %	4000

*The average steady states for the set of examined models and the associated standard deviations as well as the SD/average ratios are summary statistics of the structural uncertainty
The range within the brackets represents the 95% CI when the parameter uncertainty regarding all the model parameters is considered

4 Discussion

This paper provides an approach for modelling simultaneous yield response to P and N inputs as well as to plant-available soil test P in crop production. The modelling approach was demonstrated by developing yield response models primarily for the purpose of dynamic economic analysis, including both nutrients as decision variables and STP as a state variable. This study therefore contributes to the rich body of literature regarding separate yield responses to main nutrients. Prior to applying the empirical data to the model estimation, several assumptions had to be made given the separate datasets and the specific objectives of the model. These assumptions led to the applied multiplicative structure of the yield response model (illustrated in [1]), which resembles the Mitschelich-Baule response function presented by Frank et al. (1990). In addition, the applied iterative approach to model building process is often suggested (e.g., Refsgaard et al. 2007; Bennett et al., 2013). The structure of the model and the modelling process can be generally applied for any region, given that the required datasets are available.

The approach, in which separate datasets are applied for estimation, is a convenient way to avoid the problem of multicollinearity, which is known to be a prominent problem when attempting to estimate the effect of a certain nutrient from a compound fertiliser experiment data (e.g. Sheahan et al., 2013). The disadvantage of exploitation of separate datasets, however, is that the possible interaction effect of N and P fertilisers cannot be captured from the datasets regarding field experimentation where only one nutrient has been varied at a time (Sumner and Farina, 1986). In some cases fertilisation with P is shown to increase N uptake by plants (e.g., Black and Wight, 1979; Graciano et al., 2006). Nevertheless, since the models performed relatively well within the NPK-fertiliser field experiment datasets utilised in validation, the validation results suggest that the interaction of N and P fertilisers might be a less important factor than the individual effects of N, P and STP.

The validation results also suggested that the integrated models for both soil textures could predict the observation patterns successfully and without considerable bias. Thus, the applied model structure and the underlying assumptions appeared to be appropriate, although the fitted multiple functional forms predicted the observations of the initial dataset with varying degrees of success. The datasets for the first model element in [1] were clearly the scarcest and of worse quality compared to the other datasets. As a result, we could not fit a model that would have explained the data variation regarding clay soils. Thus, our results suggest that other factors might be more important than STP in determining the level of P-control yield, particularly for clay soils. Nevertheless, since the correlation between STP and P-control yield was significant in the initial dataset, we could conclude that the dataset did not support the function form satisfying the initial assumptions, particularly the concavity of the yield response curve. The concave function form, however, was essential for the successful model performance in the dynamic economic optimisation, which was the primary application for what the models were build for.

Our findings suggest that there was no considerable uncertainty regarding the parameter estimates. However, as was noted by Wallach et al. (2006, p.222), when the model elements are multiplied with each other in complex system models, even small uncertainties related to model parameters may have a notable effect on model output when the parameters get mixed together and various interactions take place. The multiplication of uncertainties may be considered a disadvantage of the applied multiplicative model structure. The results of the DSA and the Monte Carlo analysis showed that the parameter uncertainty greatly affected the economic optimums in the case of clay soils, whereas the effect was relatively minor for coarse soils. This was expected because there was more uncertainty related to the datasets for clay soils and the effect of parameter uncertainty on economic optimums clearly stemmed mostly from model elements, which were estimated from the uncertain datasets. Also Briggs et al. (2012) noticed a positive association between uncertain datasets and the parametric uncertainty. In addition, for both soil textures the parameter uncertainty had a greater effect on the optimal model output than the structural uncertainty. Nevertheless, the structural uncertainty did have a greater effect on economic optimums for clay soils than for coarse soils.

Several previous studies have also indicated, similarly to our findings, the effect of a functional form choice on optimal fertiliser rates (e.g., Abraham and Rao, 1965; Anderson and Nelson, 1971; Cerrato and Blackmer, 1990). The uncertainty among the optimums clearly originated primarily from the choice of the third model element determining the N response, because the absolute amount of applied N fertiliser was greater than P fertiliser (particularly for clay soils). Similarly, Tumusiime et al. (2011) noted that the model-based optimal N rates are sensitive to structural misspecification of the yield response function. In addition, the structure of the first and second model elements was more restricted by the assumptions. Thus, there was less freedom in choosing the functional form for the first and second model element compared to the third model element.

The obtained optimal P fertilisation rates were higher for coarse soils, resulting from their higher observed P response, compared to that for clay soils. The optimal P rates were noticeably high for coarse soils, considering that the optimal STP level was fair and there was not much P response on soils characterised by such STP levels. However, for coarse soils, the yield reached its maximum at higher STP levels than for clay soils (Appendix 3, Fig. 1a, b). Therefore, for coarse soils, with the given price ratio and discount rate (see Fig. 8), it was optimal to maintain fair STP class throughout the planning horizon. Because of the annual crop-uptake, this requires relatively high annual P doze (>20 kg ha^{-1}). Such optimum is in contradiction to previous results stating that for increasing STP levels yield increases from annual P application become negligible and no more economic profit is obtainable (e.g., Cox, 1992; Valkama et al., 2011). In practise, such behaviour is observed; there are farmers who prefer high P rates on fair (or even better) soils in order to secure sufficient P availability for crops (Reijneveld and Oenema, 2012; Dodd and Mallarino, 2005). In addition, it has been shown that maintaining fair or satisfactory STP levels requires surplus P fertilisation (e.g., Cox et al., 1981; Cope, 1981; Cox, 1992; Saarela et al. 2004; Dodd and Mallarino, 2005). Our results illustrate these empirical findings.

For the clay soils, on the other hand, the obtained economic optimum was consistent with suggestions by Cox (1992) and Valkama et al. (2011). The obtained optimal N rates were considerably higher for clay soils than for coarse soils. This results from the estimated response models and the respective datasets, since the observed N response was clearly higher for clay soils than for coarse soils. Thus, based on our findings, the optimal fertilisation rates for N and P depend on the soil texture. Consequently, the economic alternative costs of the maximum permissible N and P rates denoted by the agri-environmental programmes, such as the FAEP, may also depend on the soil texture of a production area. Our results suggest that the FAEP would not limit N fertilisation on coarse soils and P fertilisation on clay soils, whereas the limiting effect is notable for P fertilisation on coarse soils and for N fertilisation on clay soils.

The sensitivity analysis showed that the increase in price for N and P fertilisers decreased the optimal amount of every variable of the model whereas the increase price of barley yield increased the optimal amount of every variable. There were, however, differences in the sensitivity of the optimums for changes in these parameters across the soil textures. This originated from the different shapes of the yield response curves and surfaces for P, N and STP, which in turn affected the output and input demand elasticity of the production for clay and coarse soils (see Appendix 5). The discount rate also had a (relatively minor) decreasing effect on all the model variables. Instead, the N control yield had a considerable effect on optimal fertilisation rates, particularly on optimal N fertilisation rate, and the optimal annual yield. The effect of N control yield was greater for clay soils compared to that for coarse soils, which resulted from the fact that for clay soils the N was relatively more important factor than for coarse soils.

We identified four suggestions for further study: (1) in order to explicitly examine the relationship between STP and crop yield, more long-term data are needed, which was also suggested by Dodd et al. (2012). (2) To improve the developed models, dynamics of soil N and SOM should be taken into account because it is well known that the N status in the soil is quite dynamic (e.g., Karlsson, 2012; Baoqing et al., 2014; Wild et al., 2015). Instead, we included the N-control yield as a parameter which, however, does not directly reflect the N or SOM stocks of the soil. We utilised such strategy because there are no transition functions for N and SOM (comparable to those regarding P), which could be coupled with the estimated yield response functions to enable the proper inclusion of the soil's N and SOM dynamics into the system model. (3) Introducing weather-related variables would reduce the model output uncertainty since the seasonal variability could be taken into account. According to previous studies, the annual weather-related variation in production conditions often has a stronger effect on annual yields than the fertilisation application rates (e.g., Nemeth, 2006; Valkama et al., 2011; Salo et al., 2013). We attempted to eliminate this effect by applying weighted estimation methods. Introducing weather-related variables would result in a stochastic model rather than deterministic, which would be more realistic since agriculture is strongly influenced by seasonal weather- and climate-related uncertainty (e.g., Tumusiime et al. 2011; Boyer et al., 2013; Ray et al., 2015). (4) The explicit examination of the leaching potential remains as a subject of further studies, since this would require that the system models were coupled with appropriate leaching functions for N and P.

5 Conclusions

We discovered that the evaluation of a model performance in multiple steps is a valid strategy in recognising the best models and in evaluating the amount of parameter and structural uncertainty, as well as the effect of uncertainty on economic optimums. Our findings showed that when dynamic aspects are taken into account in economic optimisation, there might be conditions under which it is reasonable to maintain fair soil P levels by high annual P rates, although the relative annual response to P fertilisation is low on such soils. Consequently, it might be insufficient to base conclusions regarding profitable fertiliser application on a short-term analysis, as is the case when dynamic feedbacks relating to soil phosphorus are ignored. Further, based on our results, the optimal fertilisation rates for N and P, as well as the economic alternative costs of the maximum permissible N and P rates denoted by the agri-environmental programmes, appear to be soil texture-depended. Therefore, the efficiency of such programmes could be improved by targeting different fertilisation limits for different soil textures. The developed yield response models can also be coupled with leaching functions and extended for economic analysis under changing environmental conditions. In addition, although the estimated model structures and parameters represent Finnish soil quality and production conditions, the applied modelling approach as well as the multiplicative model structure can be generalised to any other region, given that adequate sets of empirical data are available.

6 Acknowledgements

This work resulted from the BONUS BALTICAPP project and was supported by BONUS (Art 185), funded jointly by the EU and Academy of Finland. We would like to thank Antti Iho, Antonios Rezitis, Timo Sipiläinen and Stefan Bäckman for their constructive comments. All remaining errors are the authors' responsibility.

7 References

Abraham, T.P., and Rao, V.Y., (1965): An investigation on functional models for fertilizer response studies. J. Indian Soc. Agric. Stat. 18, pp.45-61

Aigner, D., Lovell, C.A.K., and Schmidt, P., (1977): Formulation and estimation of stochastic frontier production function models. Journal of Econometrics 6, pp.21-37

Akaike, H., (1973): Information theory as an extension of the maximum likelihood principle. Pages 267-281 in B.N. Petrov and F. Csaki, editors. Second International Symposium on Information Theory. Akademiai Kiado, Budapest.

Alley, M.M., Martz, Jr.M.E., Davis, P.H., and Hammons, J.L., (2009): Nitrogen and Phosphorus Fertilization of Corn. Virginia Cooperative Extension Publication 424-027

Analogides, D., and Rendig, V.V., (1972): Functional Relationships between Yield Response and Soil Phosphorus Supply, Choice of the independent variable. Plant and Soil 37, pp.545-559

Anderson, R.L., and Nelson, L.A., (1971): Some problems in estimation of single nutrient functions. Bulletin of the International Statistical Institute 44 (1), pp.203-222

Baoqing, C., Enke, L., Qizhuo, T., Changrong, Y., and Yanqing, Z., (2014): Soil nitrogen dynamics and crop residues. A review. Agronomy for Sustainable Development, Springer Verlag/EDPSciences/INRA, Volume, 34 (2), pp.429-442.

Barrow, N.J., (1980): Evaluation and Utilization of Residual Phosphorus in Soils. In book The Role of Phosphorus in Agriculture by Khasawneh F.E., Sample E.C., Kanprath E.J., Huffman E.O., Olsen S.R., Pope A., Stelly M., and Dinaue R.C.,. Published by American Society of Agronomy, Crop Science Society of America, Soil Science Society of America, Madison, Wisconsin USA 1980.

Bellman, R., (1957): Dynamic Programming. Princeton University Press, Princeton, New Jersey. 339p.

Bennett, N.D., Croke, B.F.W., Guariso, G., Guillaume, J.H.A., Hamilton, S.H., Jakeman, A.J., Marsili-Libelli, S., Newham, L.T.H., Norton, J.P., Perrin, C., Pierce, S.A., Robson, B., Seppelt, R., Voinov, A.A., Fath, B.D., and Andreassian, V., (2013): Characterising performance of environmental models. Environmental Modelling and Software 40 (2013), pp.1-20

Black, A.L., and Wight, R.J., (1979): Range Fertilization: Nitrogen and Phosphorus Uptake and Recovery Over Time. Journal of range management 32(5), pp.349-353

Bock, B.R., and Sikora, F.J., (1990): Modified-Quadratic/Plateau Model for Describing Plant Responses to Fertilizer. Soil Sci. Soc. Am. J. 54, pp.1784-1789

Bolland, M.D.A., Allen, D.G., and Barroe, N.J., (2003): Sorption of phosphorus by soils: how it is measured in Western Australia. Department of Agriculture and Food, Western Australia. Bulletin 4591, 32p

Boyer, C.N., Larson, J.A., Roberts, R.K. McClure, A.T., Tyler, D.D., and Zhou, V., (2013): Stochastic Corn Yield Response Functions to Nitrogen for Corn after Corn, Corn after Cotton, and Corn after Soybeans. Journal of Agricultural and Applied Economics, 45,4 (November 2013), pp.669–681

Briggs, A.H., Weinstein, M.C., Fenwick, E.A.L., Karnon, J., Sculpher, M.J., and Paltiel, A.D., (2012): Model Parameter Estimation and Uncertainty: A Report of the ISPOR-SMDM Modeling Good Research Practices Task Force-6. Value in health 15 (2012), pp.835–842

Burnham, K.P., and Anderson, D.R., (2002): Model Selection and Multimodel Inference: A practical information-theoretic approach. Second Edition. Springer

Busman, L., Lamb, J., Randall, G., Rehm, G., and Schmitt, M., (2002): The nature of phosphorus in soils. Copyright © 2002 Regents of the University of Minnesota. All rights reserved. Reviewed July 2009

Bäckman, S., and Lansink, A.O., (2005). Crop and soil specific N and P efficiency and productivity in Finland. Agriculture and food science. Vol 14 (2005), pp.264-276.

Carlgren, K., and Mattson, L., (2001): Swedish Soil Fertility Experiments. Acta Agriculture Scandinavica, Sect. B, Soil and Plant Sci. 2001: 51, pp.49-76

Cerrato, M.E., and Blackmer, A.M., (1990): Comparison of models for describing corn yield response to nitrogen fertilizer. Agronomy Journal 82, pp.138-143.

Cope, J.T., (1981): Effects of 50 years of fertilization with phosphorus and potassium on soil test levels and

366 yields at six locations. Soil Sci. Soc. Am. J. 45,pp.342-347.

Cox, F.R., Kamprath, E.J., and McCollum, R.E., (1981): A descriptive model of soil test nutrient levels following

369 fertilization. Soil Sci. Soc. Am. J. 45, pp.529-532.

Cox, F.R., (1992): Range in soil phosphorus critical levels with time. Soil Sci. Soc. Am. J. 56, pp.1504-1509.

Dodd, J.R., and Mallarino, A.P., (2005): Soil-test Phosphorus and Crop Grain Yield Responses to Long-Term Phosphorus Fertilization for Crop-Soybean Rotations. Soil Sci. Soc. Am. J. 69, pp.1118-1128

Dodd, R.J., McDowell, R.W., and Condron, L.M., (2012): Predicting the changes in environmentally and agronomically significant phosphorus forms following cessation of phosphorus fertiliser applications to grassland. Soil Use and Management 28, pp.135-147

Eckersten, H., Jansson, P.-E., and Johnsson, H., (1994): SOILN model – user's manual 2nd edition, Division of Agricultural Hydrotechnics Communications 94:4, Department of Soil Sciences, Swedish University of AgriculturalSciences, 58pp, Uppsala.

Engelstad, O.P., and Teraman, G.L., (1980): Agronomic Effectiveness of Phosphate Fertilizers. In book The Role of Phosphorus in Agriculture by Khasawneh, F.E., Sample, E.C., Kanprath E.J., Huffman, E.O., Olsen, S.R., Pope, A., Stelly, M., and Dinaue, R.C.,. Published by American Society of Agronomy, Crop Science Society of America, Soil Science Society of America, Madison, Wisconsin USA 1980.

Esala, M., and Larpes, G., (1984): Kevätviljojen sijoituslannoitus savimailla. MTTK – Maatalouden Tutkimuskeskus, tiedote 2/84. Maanviljelyskemian ja –fysiikan osasto, Jokioinen 1984, ISSN 0359-7652

Frank, M.D., Beattie B.R., and Embleton, M.E., (1990): A Comparison of Alternative Crop Response Models. American Journal Of Agricultural Economics, 1990 Aug, Vol.72(3), pp.597-603

Gilbert, N., (2009): The disappearing nutrient. Nature, 461, pp.716–718

Gornall, J., Betts, R., Burke, E., Clark, R., Camp, J., Willett, K., and Wiltshire, A., (2010): Review, Implications of climate change for agriculture productivity in the early twenty-first century. Phil. Trans. R. Soc. B (2010) 365, pp.2973-2989

Graciano, C., Goya, J.F., Frangi, J.L., and Guiamet, J.J., (2006): Fertilization with phosphorus increases soil nitrogen absoption in young plants of Eucalyptus grandis. Forest Ecology and Management 236 (2006) 202-210

Griffin, R.C., Montgomerym, J.M., and Rister, M.E., (1987): Selecting Functional Form in Production Function Analysis. Western Journal of Agricultural Economics, 12(2), pp.216-227

Griffith, W.K., (1992): Crop responses at high soil test phosphorus levels. Better Crops/Fail 1992, pp.12-15

Hansen, S., Jensen, H. E., Nielsen, N. E. and Svendsen, H. (1990): DAISY: A Soil Plant System Model. Danish Simulation Model for Transformation and Transport of Energyand Matter in the Soil-Plant-Atmosphere System. Copenhagen: National Agency for Environmental Protection.
Hardaker, J.B., Gudbrand, L., Anderson, J.R., and Huirne, R.B.M., (2015): Coping with Risk in Agriculture, 3rd Edition. Applied Decision Analysis.
Hildreth, C.G., (1955): Discrete models with qualitative descriptions. Methodological procedures in the economic analysis of fertilizer use data. Ed. E.L. Baum, E.O. Heady, and J. Blackmore, pp. 62-75. Ames: Iowa State College Press, 1955
Hooda, P., Truesdale, V., Edwards, A., Withers, P., Aitken, M., Miller, A. and Rendell, A., (2001): Manuring and Fertilization Effects on Phosphorus Accumulation in Soils and Potential Environmental Implications. *Advances in Environmental Research* 5: 13–21.
Iho, A., and Laukkanen, M., (2012): Dynamically Optimal Phosphorus Management and Agricultural Water Protection. MTT Discussion papers 4/2009
Iho, A., and Laukkanen, M., (2012): Precision phosphorus management and agricultural phosphorus loading. Ecological Economics. Volume 77, May 2012, pp.91–102 http://www.sciencedirect.com/science/article/pii/S0921800912000742
Judel, G.K., Gebauer, W.G. and Mengel, K., (1985): Yield response and availability of various phosphate fertilizer types as estimated by EUF. Plant and Soil 83, pp.107-115 (1985). Martinus Ni/hoff/Dr W. Junk Publishers, Dordrecht. Printed in the Netherlands.
Just, R.E., and Weninger, Q., (1999): Are Crop Yields Normally Distributed? American Journal of Agricultural Economics. Vol. 81, No. 2 (May, 1999), pp. 287-304
Kang, Y., Khan, S., and Xiaoyi, M., (2009): Climate change impacts on crop yield, crop water productivity and food security – A review. Progress in Natural Science. Volume 19, Issue 12, 10 December 2009, pp.1665–1674
Karlsson, T., (2012): Carbon and Nitrogen Dynamics in Agricultural Soils Model Applications at Different Scales in Time and Space. Doctoral Thesis, Swedish University of Agricultural Sciences, Uppsala 2012
Keating, B. A., Carberry, P. S., Hammer, G. L., Probert, M. E., Robertson, M. J., Holzworth, D., Huth, N. I., Hargreaves, J. N. G., Meinke, H., Hochman, Z., Mclean, G., Verburg, K., Snow, V., Dimes, J. P., Silburn, M., Wang, E., Brown, S., Bristow, K. L., Asseng, S., Chapman, S., Mccown, R. L., Freebairn, D.M. and Smith, C. J., (2003): An overview of APSIM, a model designed for farming systems simulation. European Journal of Agronomy 18, pp.267–288.
Mallarino, A.P., and Prater, J., (2007): Corn and soybean grain yield, phosphorus removal, and soil-test responses to long-term phosphorus fertilization strategies. 2007 Integrated Crop Management Conference - Iowa State University, pp.241-254
McLaughlin, M.J., Alston, A.M., and Martin, J.K., (1988): Phosphorus cyclin in wheat-pasture rotations. I. The source of phosphorus taken up by wheat. Aust J Soil Res 26: pp.323-331
Meeusen, W., and Broeck, J. van den (1977): Efficiency estimation from Cobb-Douglas production functions with composed error. International Economic Review 18: pp.435-444
Ministry of Agriculture and Forestry, (2014): Government regulation of environmental compensation. https:// www.finlex.fi/fi/laki/alkup/2015/20150235. Web address verified 02 02.2017 (In Finnish).
Myyrä, S., Pietola, K., and Yli-Halla, M., (2007): Exploring long-term land improvements under land tenure insecurity. Argricultural Systems 92 (2007), pp.63-75.
Nemeth, T., (2006): Application of the Bray-Mitscherlich equation approach for economically and environmentally sound fertilization of field crops in Hungary. Communications in Soil Science and Plant Analysis, 37, pp.15-20, 2227-2247, DOI:10.1080/00103620600817556
Niklas, K.J., (2006): Maximum plant height and the biophysical factors that limit it. Tree Physiology 27, pp.433–440, © 2007 Heron Publishing—Victoria, Canada
Popp, A., Calvin, K., Fujimori, S., Havlik, P., Humpenöder, F., Stehfest, E., Bodirsky, B.L., Dietrich, J.P., Doelmann, J.C., Gusti, M., Hasegawa, T., Kyle, P., Obersteiner, M., Tabeau, A., Takahashi, K., Valin, H., Waldhoff, S., Weindl, I., Wise, M., Kriegler, E., Lotze-Campen, H., Fricko, O., Riahi, K., and van Vuuren, D.P., (2016): Land-use futures in the shared socio-economic pathways. Global Environmental Change 42 (2017), pp.331-345
Ray, D.K., Gerber, J.S., Graham, K., MacDonald, and West, P.C., (2015): Climate variation explains a third of global crop yield variability. Nature Communications 6, Article number: 5989 (2015). doi:10.1038/ncomms6989

Reetz, H.F.Jr., (2016): Fertilizers and their Efficient Use. International Fertilizer Industry Association (IFA) Paris, France, 2016. Copyright 2016 IFA. All rights reserved ISBN 979-10-92366-04-4. The publication can be downloaded from IFA's website.

Refsgaard, J.C., van der Sluijs, J.P., Hojberg, A.L., and Vanrolleghem, P.A., (2007): Uncertainty in the environmental modelling process - A framework and guidance. Environmental Modelling & Software 22 (2007) 1543-1556

Reijneveld, A., and O., Oenema. (2012): Developments in soil phosphorus status in a recently reclaimed polder in the Netherlands. Nutr. Cycl. Agroecosyst. 94: pp.33-45.

Roberts, T.L., (2008): Improving Nutrient Use Efficiency. Turk J Agric For 32 (2008), pp.177-182

Rötter, R.P., Palosuo, T., Kersebaum, K.C., Angulo, C., Bindi, M., Ewert, F., Ferrise, R., Hlavinka, P., Moriondo, M., Nendel, C., Olesen, J.E., Patil, R.H., Ruget, F., Takac, J., and Trnka, M., (2012): Simulation of spring barley yield in different climatic zones of Northern and Central Europe: A comparison of nine crop models. Field Crops Research 133 (2012), pp.23-36.

Saarela, I., Järvi, A., Hakkola, H., and Rinne, K., (1995): Fosforilannoituksen porraskokeet 1977 – 1994. Vuosittain annetun fosforimäärän vaikutus maan viljavuuteen ja peltokasvien satoon monivuotisissa kenttäkokeissa. MAATALOUDEN TUTKIMUSKESKUS TIEDOTE 16/95

Saarela, I., Järvi, A., Hakkola, H., and Rinne, K., (2004): Phosphorus status of diverse soils in Finland as influenced by long-term P fertilisation 2. Changes of soil test values in relation to P balance with references to incoroporation depth of residual and freshly applied P. Agricultural and food silence. Vol. 13 (2004), pp.276-294

Salo, T., Turtola, E., Virkajärvi, P., Saarijärvi, K., Kuisma, P., Tuomisto, J., Muurinen, S., and Turakainen, M., (2013): Nitrogen fertilizer rates, N balances, and related risk of N leaching in Finnish agriculture. MTT Report 102. ISBN 978-952-487-462-5. ISSN 1798-6419. http://www.mtt.fi/mttraportti/pdf/mttraportti102.pdf

Sargent, R.G., (2011): Verification and validation of simulation models. Proceedings of the 2011 Winter Simulation Conference S. Jain, R.R. Creasey, J. Himmelspach, K.P. White, and M. Fu, eds. 978-1-4577-2109-0/11/$26.00 ©2011 IEEE, pp.183-198

Sharpley, A.N., (1986): Disposition of fertilizer phosphorus applied to winter wheat. Soil Sci Soc Am J 50: pp.953-958

Sharpley, A.N., Chapra, S.C., Wedepohl, R., Sims, J.T., Daniel, T.C. and Reddy, K.R., (1994): Managing agricultural phosphorus for protection of surface waters: issues and opinions. Journal of Environmental Quality, 23, pp.437–451.

Sheahan, M., Black, R., and Jayne, T.S., (2013): Are Kenyan Farmers Under-Utilizing Fertilizer? Implications for Input Intensification Strategies and Reseach. Food Policy 41, pp.39-52

Snyder, C.S. (2006): Best Management Practices for Fertilizer. A regional newsletter published by the Potash & Phosphate Institute (PPI) and the Potash & Phosphate Institute of Canada (PPIC)

Spiess, A-N., and Neumeyer, N., (2010): An evaluation of R2 as an inadequate measure for nonlinear models in pharmacological and biochemical research: a Monte Carlo approach. BMC Pharmacology 2010, 10:6, pp.1471-2210

Strauss, F., Schmid, E., Moltchanova, E., Formayer, H., and Wang, X., (2012): Modeling climate change and biophysical impacts of crop production in the Austrian Marchfeld Region. Climatic Change 111, 641–664.

Sumner, M.E., and Farina, M.P.W., (1986): Phosphorus Interactions with Other Nutrients and Lime in Field Cropping Systems. In: Stewart B.A. (eds) Advances in Soil Science. Advances in Soil Science, vol 5. Springer, New York, NY

Tumusiime, E., Brorsen, B.W., Mosali, J., Johnson, J., Locke, J., and Biermacher, J.T., (2011): Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models. Journal of Agricultural and Applied Economics, 43,4(November 2011), pp.541-552

Uusitalo, R., Hyväluoma, J., Valkama, E., Ketoja, E., Vaahtoranta, A., Virkajärvi, P., Grönroos, J., Lemola, R., Ylivainio, K., Rasa, K., and Turtola, E., (2016): A Simple Dynamic Model of Soil Test Phosphorus Responses to Phosphorus Balances. J Environ Qual. 2016 May; 45(3), pp.977-83

Valkama, E., Uusitalo, R., Ylivainio, K., Virkajärvi P., and Turtola E., (2009): Phosphorus fertilization: A meta-analysis of 80 years of reseach in Finland. Agriculture ecosystems & environment 130 (3), pp.75-85

Valkama, E., Uusital,o R., and Turtola, E., (2011): Yield response models to phosphorus application: A research synthesis of Finnish field trials to optimize fertilizer P use of cereals. Nutrient Cycling in Agroecosystems, (2011) 91:1-15

Valkama, E., Salo T., and Esala, M., (2013): Nitrogen balances and yields of spring cereals as affected by nitrogen fertilization in northern conditions: A meta-analysis. Agriculture, Ecosystems and Environment 164 (2013), pp.1– 13.

Van der Velde, M., Folberth, C., Balkovič, J., Ciais, P., Fritz, S., Janssens, I. A., Obersteine, M., See, L., Skalský, R., Xiong, W., Peñuelas, J., (2013): African crop yield reductions due to increasingly unbalanced Nitrogen and Phosphorus consumption, Global Change Biology (2014), 20, pp.1278–1288

Wallach, D., Makowski, D., Jones, J.W., and Brun, F., (2006): Working with dynamic crop models. Methods, tools, and examples for agriculture and environment. Elsevier B.V.

Wild, B., Schnecker, J., Knoltsch, A., Takriti, M., Mooshammer, M., Gentsch, N., Mikutta, R., Alves, R.J.E., Gittel, A., Lashchinskiy, N., and Richter, A., (2015): Microbial nitrogen dynamics in organic and mineral soil horizons along a latitudinal transect in western Siberia, Global Biogeochem. Cycles, 29, pp.567–582

Xiaofei, Li., Coble K.H., Tack J.B., and Barnett B.J., (2016): Estimating Site-Specific Crop Yield Response using Varying Coefficient Models. Selected Paper prepared for presentation at the 2016 Agricultural & Applied Economics Association Annual Meeting, Boston, Massachusetts, July 31-August 2, 30 pages

Yajragupta, Y., Halley, L. E. and Melsted, S. W., (1963): Correlation of phosphorus soil test values with riee yields in Thailand. Soil Sei Soe. Am. Proe. 27, pp.395-398

Ylivainio, K., Sarvi, M., Lemola, R., Uusitalo, R., and Turtola, E., (2014): Regional P stocks in soil and in animal manure as compared to P requirement of plants in Finland. MTT Report 124.

8 Appendix

Structure of the Appendix section is the following:

Appendix 1: Description of the database
Appendix 2: Standardisation of the separate data sets
Appendix 3: Results of the model estimation and the model selection
Appendix 4: Model evaluation
4.1. Clay soils models
4.2. Coarse soils models
Appendix 5: Analytical properties of the system models
5.1. Analytical properties of the transition models
Appendix 6: Analytical properties of the estimated yield response models
6.1. Marginal productivity of the P and N fertilisers and the STP
6.2. Isoquants
6.3. Elasticity of the yield with respect to the P and N fertilisers
Appendix 7: Results of the Monte Carlo analysis
Appendix 8: Results of the sensitivity analysis
Appendix 9: References for appendix

Appendix 1: Description of the database

Table 1
Descriptive statistics of the dataset consisting of P fertilisation experiments for coarse and clay soils.

Soil texture	Variable	min	max	mean	std	CV
Coarse	P fertilisation (kg ha^{-1})	0	72	25.27	20.41	152 %
Coarse	STP (mg P l^{-1})*	3.7	49.8	14.72	13.44	91 %
Coarse	Yield response (ratio)	0.93	1.4	1.084	0.1175	10.8 %
n=66, Source: Valkama et al. 2011						
Soil texture	Variable	min	max	mean	std	CV
Clay	P fertilisation (kg ha^{-1})	0	100	23.24	21.74	94 %
Clay	STP (mg P l^{-1})*	0.8	40	11.15	12.08	108 %
Clay	Yield response (ratio)	0.96	1.19	1.046	0.0609	5.8 %
n=42, Source: Valkama et al. 2011.						
* STP was measured at the beginning of experiments from the control (without added P)						

Table 2
Descriptive statistics of the dataset consisting of P fertilisation experiment results without added P

Soil texture	Variable	min	max	mean	std	CV
Coarse	STP (mg P l^{-1})*	3	49.8	12.83	12.45	97 %
Coarse	P-Control yield (kg ha^{-1})	1730	6000	3222	1200	37 %
n=21, Source: Valkama et al. 2011 and Saarela et al. 1995						
Soil texture	Variable	min	max	mean	std	CV
Clay	STP (mg P l^{-1})*	0.8	50.5	12.93	15.37	119 %
Clay	P-Control yield (kg ha^{-1})	2000	4749	3700	730	19.7 %

n=18, Source: Valkama et al. 2011 and Saarela et al. 1995
* STP was measured at the beginning of experiments from the control (without added P)

Table 3
Descriptive statistics of the dataset consisting of N fertilisation experiments on coarse and clay soils.

Soil texture	Variable	min	max	mean	std	CV
Coarse	N fertilisation (kg ha^{-1})	16	216	77.73	39.72	51 %
Coarse	N-control yield kg ha^{-1})	1000	4500	2496	869.4	35 %
Coarse	Yield response (ratio)	1.04	2.15	1.533	0.2802	18 %

n=63, Source: Valkama et al. 2013

Soil texture	Variable	min	max	mean	std	CV
Clay	N fertilisation (kg ha^{-1})	16	180	97.94	38.36	39 %
Clay	N-control yield (kg ha^{-1})	1700	3900	2279	359.6	16 %
Clay	Yield response (ratio)	1.14	2.7	0.957	0.223	23 %

n=36, Source: Valkama et al. 2013

Table 4
Descriptive statistics of the validation dataset consisting of NPK fertilisation experiment results

Soil texture	Variable	min	max	mean	std	CV
Coarse	STP (mg P l^{-1})	3.9	21.6	14.63	6.818	46.6 %
Coarse	P fertilisation (kg/ha)	0	73.03	16.91	13.71	81 %
Coarse	N fertilisation (kg ha^{-1})	0	160	81.04	42.5	52 %
Coarse	Yield (kg ha^{-1})	1223	4850	3664	782.6	21.4 %

n=35

Soil texture	Variable	min	max	mean	std	CV
Clay	STP (mg P l^{-1})	3.2	31.2	11.54	6.346	55 %
Clay	P fertilisation (kg/ha)	0	109	27.98	26.5	95 %
Clay	N fertilisation (kg ha^{-1})	0	200	91.62	51.9	57 %
Clay	Yield (kg ha^{-1})	894	5194	3307	1053	32 %

n=68

Appendix 2: Standardisation of the separate data sets

In P fertiliser experiments, there was a certain amount of N fertilisation applied in addition to P fertilisation. This N rate, which was most likely considered to be sufficient for P being the limiting factor, was treated as a reference N rate to which the N data were scaled: when the applied N rate was equal to the reference N rate, the yield response in the scaled dataset remained the same as the one observed. Instead, when the applied N rate was lower than the reference N rate, the yield response in the scaled dataset was higher than the observed one. The standardisation of the separate data sets proceeded according to following steps: (1) We defined the weighted average N fertilisation rate applied in P fertilisation experiments as a reference N rate (kg ha^{-1}) and denoted it by: $N_{ref} = \frac{\sum_{i=1}^{k} n_i N_i}{\sum_{i=1}^{k} n_i}$, where n_i with $i \in \{1,..,k\}$ is a number of experiment years in a given experiment i and N_i is an associated N fertilisation rate. This rate was approximately 70 and 95 kg ha^{-1} for coarse and clay soils, respectively (it must be noted that these rates are rather low, particularly for coarse soils). (2) We determined the yield response corresponding to the reference N rate with the following relationship: $\frac{N_{ref}}{N_i} = \frac{\Delta y(N_{ref,i})}{\Delta y(N_i)} => \Delta y(N_{ref,i}) = \frac{N_{ref}}{N_i} \Delta y(N_i)$, where, $\Delta y(N_i)$ is the yield response observed in N-experiment i corresponding to N_i and $\Delta y(N_{ref,i})$ is the yield response that would be obtained by applying the reference amount of N fertilisation in this given experiment. (3) We took the average of the calculated yield responses corresponding to a reference N fertilisation rate, $\Delta \bar{y}(N_{ref})$, and we divided every yield response observed in the N experiment data by it: $\varphi_{N,i} = \Delta y(N_i)\left(\Delta \bar{y}(N_{ref})\right)^{-1}$, where $\varphi_{N,i}$ implies the scaling factor for yield response for N fertilisation in experiment i. Thus, the observed yield responses in the N experiments were scaled in such a way that the yield response corresponding to the yield response gained by applying the reference N fertilisation rate was one. In other words, the N dataset were mean-scaled with the hypothetical average N response corresponding the average N rate applied in P experiments. This scaled data were utilised for the estimation of the third model element.

Appendix 3: Results of the model estimation and the model selection

In Table 5, the estimated models are ranked from best to worst with the best model represented in bold print. Most of the models required some modification of the basic functional forms, combination of various functions or both. The applied basic functional forms were square root, Spillman, exponent, quadratic, power, logarithm, logistic and polynomial. It must be noted that we had to relax the assumption of twice-continuously differentiability (A2) for first model element in [1] for clay soils, because the dataset did not support converging functional form. Thus, we fitted a piecewise function consisting of a concave part and a plateau part (the plateau level was set to a maximum observed level ($Y_{max} = 4749$ kg ha^{-1}) for the yield and the respective STP level ($STP^{*} = 103$ mg l^{-1}) was solved from the model equation). However, since the STP^{*} is such high, the piecewise function is twice-continuously differentiability within the range of observed STP levels.

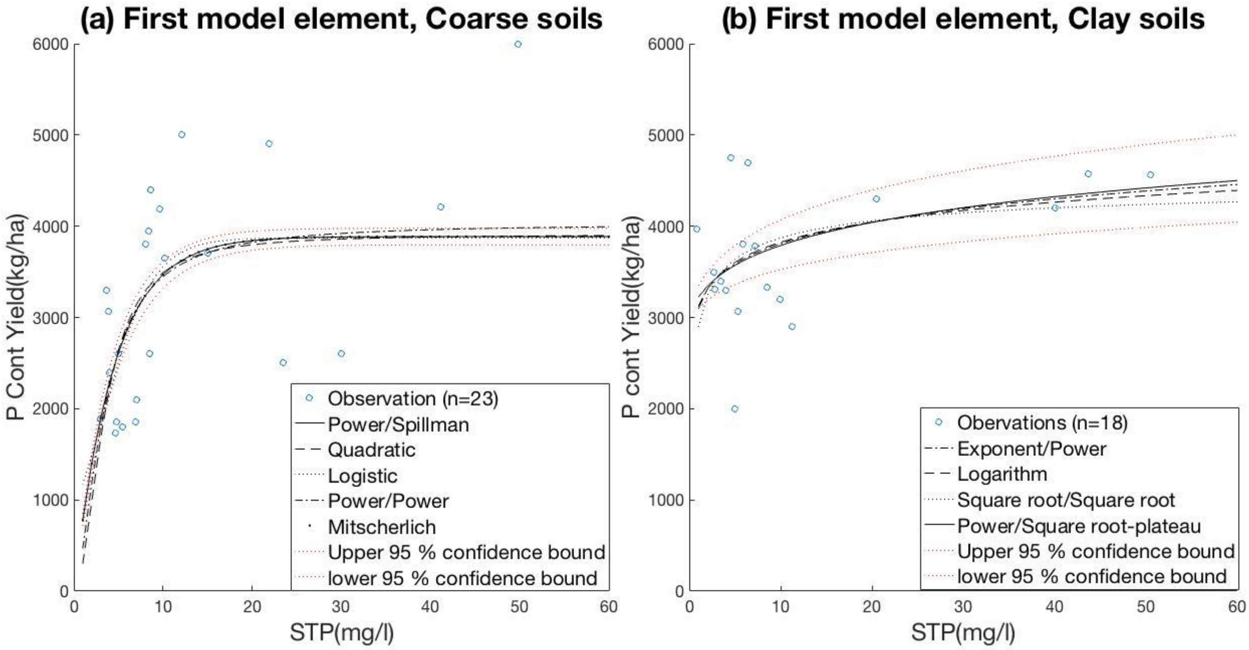
Table 5
The estimated models and the summary of the model ranking results for each model element in [1] for both soil textures*

Soil texture	Model (model element)	Model structure	***RSS***	***AICc***	*Δ*	*w*	w_1/w_j
Coarse	**Power/Spillman (1)**	$\bar{y}_{P0} = \theta_1 STP^{\frac{1}{\theta_1}}(1 - \theta_2{}^{STP})$	20.39 x10^{6}	319.587	0	0.303	1.00

Coarse	Mitscherlich (1)	$\bar{y}_{P0} = \theta_1(1 - \exp(-\theta_2 STP))$	20.39 x10[6]	319.590	0.002	0.303	1.00
Coarse	Quadratic (1)	$\bar{y}_{P0} = \theta_1 STP^2(1 + \theta_2 STP^2)^{-1}$	21.00 x10[6]	320.264	0.676	0.216	1.40
Coarse	Logistic (1)	$\bar{y}_{P0} = \theta_1\left(1 + \exp(\theta_2 STP^{\theta_3})\right)^{-1}$	19.93 x10[6]	321.723	2.136	0.104	2.91
Coarse	Power/Power (1)	$\bar{y}_{P0} = \theta_1 STP^{\frac{1}{\theta_1}}(1 + \theta_2 STP^{\theta_3})^{-1}$	20.52 x10[6]	322.392	2.804	0.075	4.06
Clay	**Power/Square root-plus-plateau (1)**	$\bar{y}_{P0} = \theta_1(STP^{\theta_2} + 1)(1 + \sqrt{STP})^{-1}$, when $STP < STP^*$ $\bar{y}_{P0} = max\bar{y}_{P0}$, when $STP \geq STP^*$	7.52 x10[6]	237.774	0	0.652	1.00
Clay	Exponent/Power (1)	$\bar{y}_{P0} = exp(\theta_1 STP^{\theta_2})$	7.83 x10[6]	238.455	0.681	0.464	1.40
Clay	Logarithm (1)	$\bar{y}_{P0} = \theta_1 \ln(\theta_2 STP + 1)$	7.99 x10[6]	238.860	1.086	0.379	1.70
Clay	Square root/Square root (1)	$\bar{y}_{P0} = \theta_1\sqrt{P}(1 + \theta_2\sqrt{STP})^{-1}$	8.81 x10[6]	240.618	2.844	0.157	4.10
Coarse	**Square root/Quadratic (2)**	$\bar{\omega}_P = \theta_1\sqrt{P}(1 + \theta_2 STP^2)^{-1} + 1$	0.276	-357.301	0	0.305	1.00
Coarse	Spillman/Quadratic (2)	$\bar{\omega}_P = \theta_1(1 - {\theta_2}^P)(1 + \theta_3 STP^2)^{-1} + 1$	0.269	-356.780	0521	0.235	1.30
Coarse	Square root/Logistic (2)	$\bar{\omega}_P = \theta_1\sqrt{P}\left(1 + exp(\theta_2 STP)\right)^{-1} + 1$	0.279	-356.592	0.709	0.214	1.43
Coarse	Power/Quadratic (2)	$\bar{\omega}_P = \theta_1 STP^{\frac{1}{\theta_1}}(1 + \theta_2 STP^2)^{-1} + 1$	0.279	-356.592	0.709	0.214	1.43
Coarse	Logarithm/Logistic (2)	$\bar{\omega}_P = \theta_1(\ln(P + 1))\left(1 + exp(\theta_2 STP)\right)^{-1} + 1$	0.30	-352.742	4.559	0.031	9.77
Clay[**]	**Square root/Logistic (2)**	$\bar{\omega}_P = \left(\theta_1\sqrt{P} + \theta_2\right)\left(1 + exp(\theta_3 STP)\right)^{-1} + \omega_{P,min}$	0.069	-262.613	0	0.434	1.00
Clay	Logarithm/Logistic (2)	$\bar{\omega}_P = \left(\theta_1(\ln(P + \theta_2))\right)\left(1 + exp(\theta_3 STP)\right)^{-1} + \omega_{P,min}$	0.070	-261.938	0.675	0.310	1.40
Clay	Mitscherlich/Logistic (2)	$\bar{\omega}_P = \left(\theta_1(1 - exp(-\theta_2 P)) + \theta_3\right)\left(1 + exp(\theta_4 STP)\right)^{-1} + \omega_{P,min}$	0.069	-260.166	2.447	0.128	3.40
Clay	Power/Logistic (2)	$\bar{\omega}_P = \left(\theta_1 P^{\theta_2} + \theta_3\right)\left(1 + exp(\theta_4 STP)\right)^{-1} + \omega_{P,min}$	0.069	-260.164	2.449	0.128	3.40
Coarse	**Logarithm/Quadratic (3)**	$\bar{\omega}_N = (\theta_1 + \theta_2(log(N + 1))^2)(1 + \theta_3 {Y_{N0}}^2)^{-1}$	0.385	-314.770	0	0.320	1.00
Coarse	Power/Quadratic (3)	$\bar{\omega}_N = \left(\theta_1 N^{\frac{1}{\theta_1}} + 1\right)(\theta_2 {Y_{N0}}^2 + \theta_3)^{-1}$	0.385	-314.666	0.104	0.303	1.05
Coarse	Square root/Quadratic (3)	$\bar{\omega}_N = (\theta_1\sqrt{N} + 1)(\theta_2 {Y_{N0}}^2 + \theta_3)^{-1}$	0.389	-314.067	0.704	0.225	1.42
Coarse	Polynomial (3)	$\bar{\omega}_N = \theta_1 + \theta_2 N + \theta_3 Y_{N0} + \theta_4 N^2 + \theta_5 N Y_{N0} + \theta_6 {Y_{N0}}^2$	0.352	-313.292	1.479	0.153	2.09
Clay	**Square root/Quadratic (3)**	$\bar{\omega}_N = (\theta_1\sqrt{N} + \theta_2 N)(\theta_3 {Y_{N0}}^2 + 1)^{-1}$	0.887	-126.587	0	0.436	1.00
Clay	Power/Quadratic (3)	$\bar{\omega}_N = \left(\theta_1 N^{\frac{1}{\theta_1}} + 1\right)(\theta_2 {Y_{N0}}^2 + \theta_3\sqrt{N})^{-1}$	0.926	-126.107	0.479	0.343	1.27
Clay	Logarithm/Quadratic (3)	$\bar{\omega}_N = (\theta_1 \log(N + 1) + 1)(\theta_2 {Y_{N0}}^2 + \theta_3)^{-1}$	0.940	-125.224	1.363	0.221	1.98

[*]RSS means sum of squared residuals, AICc means second order variant of Akaike's information criteria, Δ means AIC difference, w means Akaike's weight and w_1/w_j means evidence ratio between the best model (1) and another model j.
[**]The parameter $\omega_{P,min}$ denotes the minimum observation for yield response (0.96).

The representative yield response curves and the observed data points as well as the 95% confidence bounds for the best models are illustrated in Figures 1a and 1b. From these figures we may notice that the estimated models gave almost identical predictive yield response curves. In the case of coarse soils models, only logistic and power/power models converged outside the confidence bounds of the best model. All the curves were within the range of the confidence bounds of the best model in the case of clay soils. In addition, the models for coarse soils converged to plateau giving the maximum yield for lower STP levels than in the case of clay soils models. In addition, the P-control yield was higher for lower STP levels in the case of clay soils; the yield curves increased steeply for low STP levels and then began to converge slowly towards the maximum plateau, which was reached at high STP levels. The high variation of the observations for both soil textures is also observable from Figures 1a and 1b.



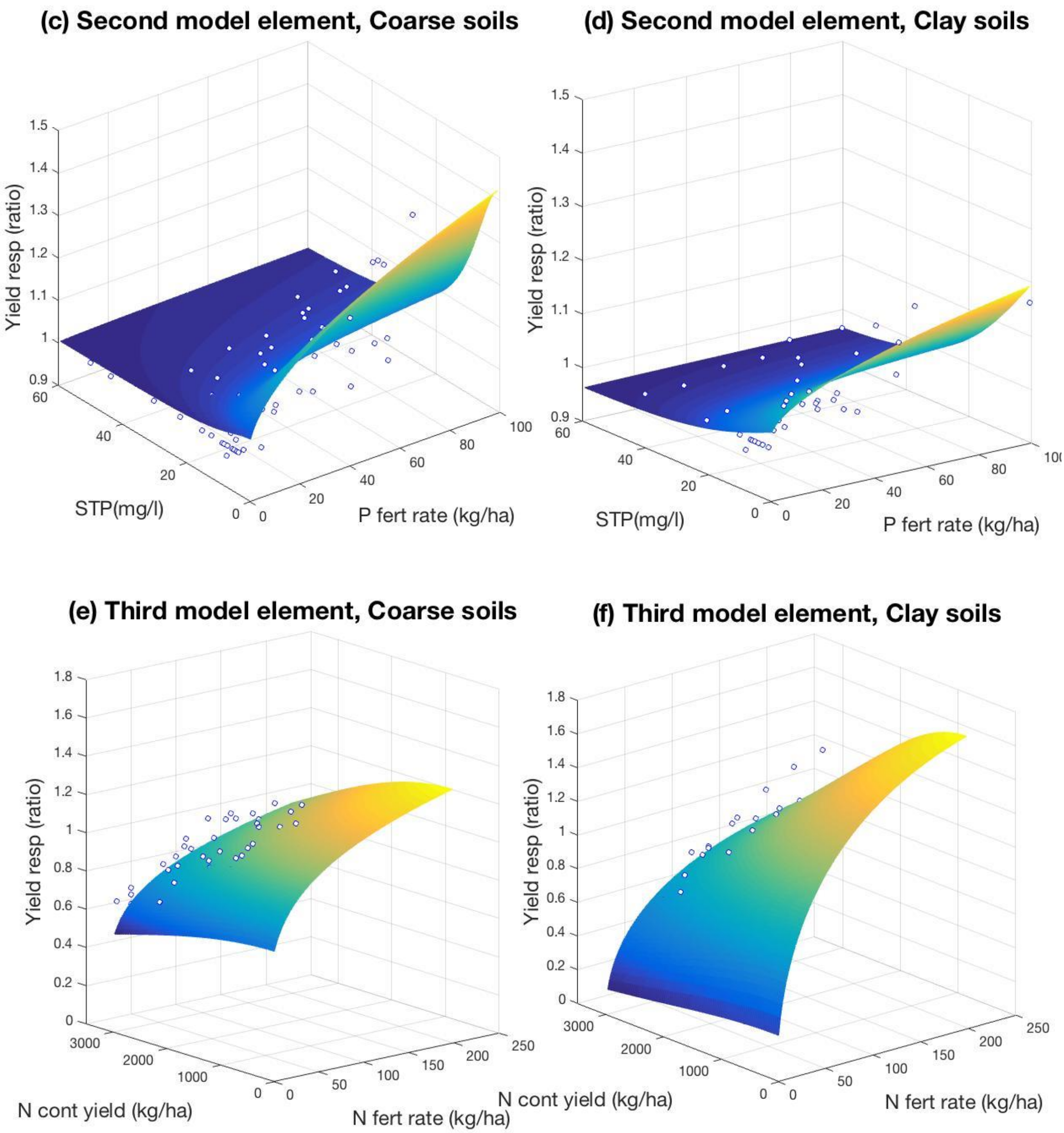


Fig. 1. The yield curves and surfaces given by the estimated models for each model element in [1] and the associated data points

Figures 1c and 1d illustrate the associated predictive response surfaces for estimated best P response-models as well as the observed data points. The yield response to P fertilisation was observed only at low STP, while it vanished for higher STP classes. The surface converged faster to its maximum plateau on clay soils, when the P rate was increased. The maximum response was also lower on clay soils. We also discovered that all the estimated models gave almost identical predictive surfaces. Figures 1e and 1f illustrate the observed data points and the response surfaces for estimated models predicting the lower yield responses to N fertilisation for higher N-control yields. The response to N fertilisation was higher for the clay soils (Fig. 1e, 1f). In addition, the coarse soils models settled for the maximum plateau for lower N rates. We also observed that the fitted forms for the N response models gave slightly different predictive surfaces, particularly in the case of clay soils. The illustrated models were the best models.

Appendix 4: Model evaluation

In this section the predictive accuracy of the estimated best models and the respected dispersion of the model residual errors is examined. The normality of the model error residuals was also examined by applying the Shapiro-Wilk normality test (Shapiro and Wilk, 1965). In addition, we did the paired t-test with null hypothesis, H_0: true difference between the means of predictions and observation is equal to 0.

4.1. Clay soils models

1. Model element (Power/square root/Plateau)

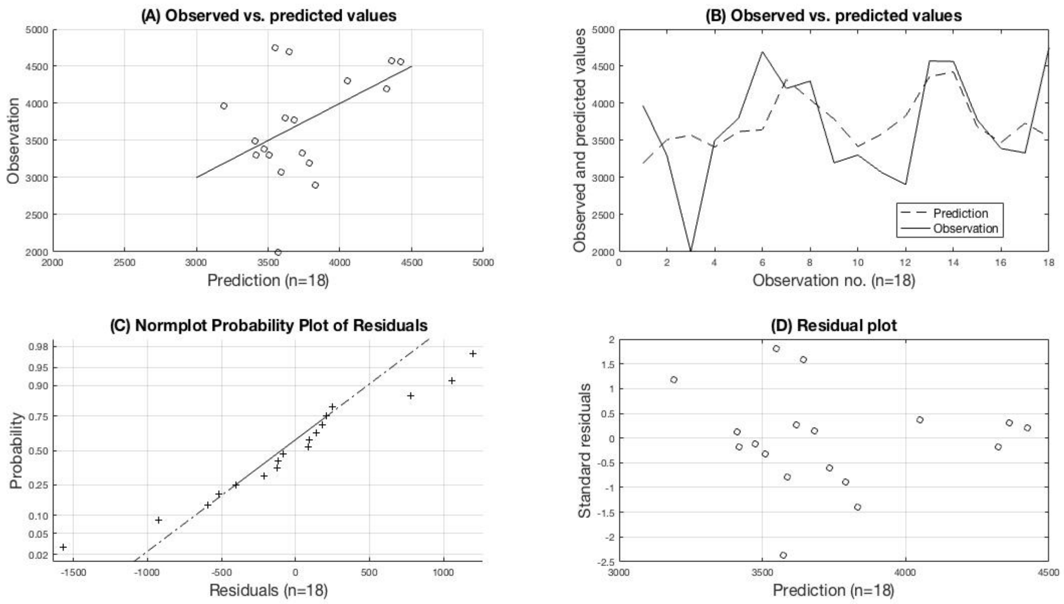


Fig. 2. Goodness-of-fit illustration of the first model element for clay soils

Shapiro-Wilk normality test: W = 0.95911, p-value = 0.5846

Paired t-test: t = 0.19381, df = 14, p-value = 0.8486

Figure 2a shows that there is no systematic bias among the model predictions. Figure 2b shows that model predicts the observations with reasonable accuracy with the exception of 1 low and 1 high observation. Figure 2c shows that these extreme observations distort the normality of the residuals. Figure 2d shows that there is no notable heteroscedasticity among the residuals.

2. Model element (Square Root/Logistic)

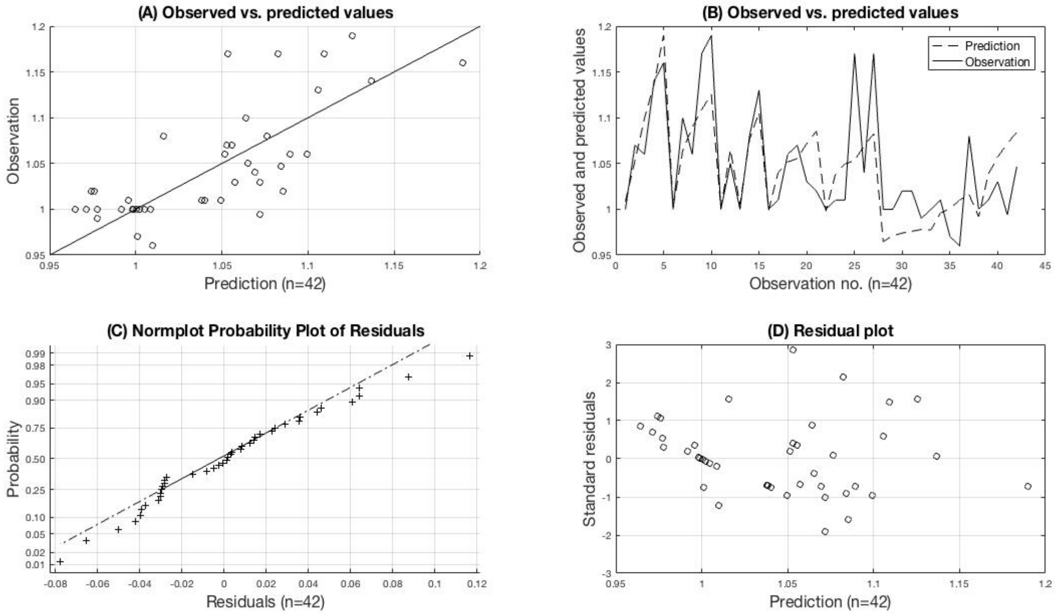


Fig. 3. Goodness of fit illustration of the second model element for clay soils

Shapiro-Wilk normality test: W = 0.97164, p-value = 0.3743

Paire t-test: t = -0.37285, df = 41, p-value = 0.7112

Figure 3a shows that the model predicts the observations without a systematic bias. Figure 3b shows that the observation patterns are predicted without a systematic bias. In addition, the residuals are normally distributed (Fig.3c) and there is no heteroscedasticity among the residuals (Fig.3d).

3. Model element (Square Root/Quadratic)

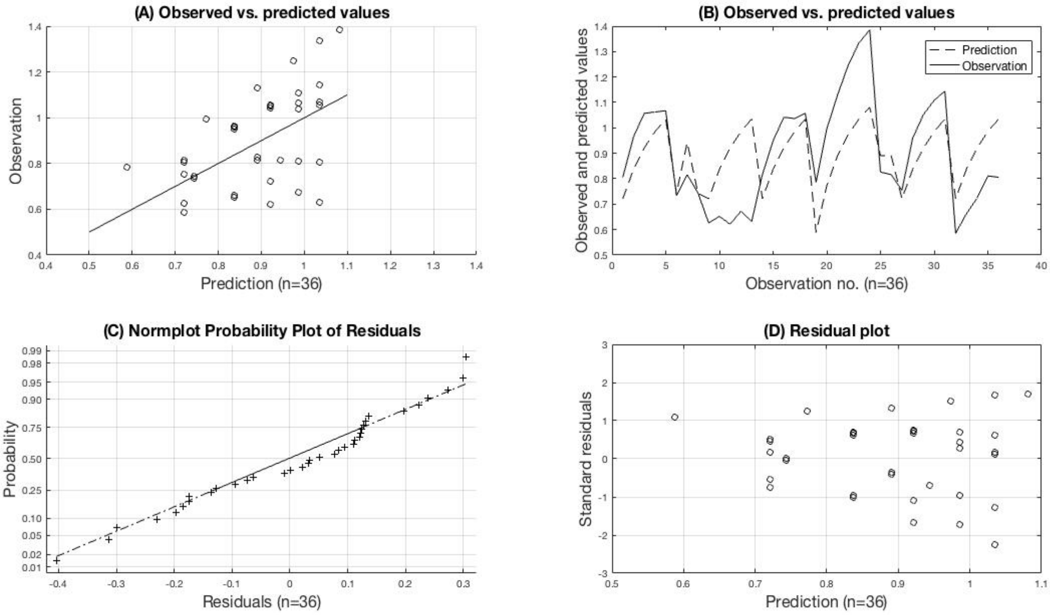


Fig. 4. Goodness of fit illustration of the third model element for clay soils

Shapiro-Wilk normality test: W = 0.9702, p-value = 0.4311

Paired t-test: t = 1.5367, df = 35, p-value = 0.1334

Figures 4a and 4d show that the variance of the predictions and the residuals increases slightly for higher predictions. Despite of this, the model predicts the observation patterns relatively accurately (Fig.4b) and the residuals are normally distributed (Fig.4c).

4.2. Coarse soils models

1. Model element (Power/Spillman)

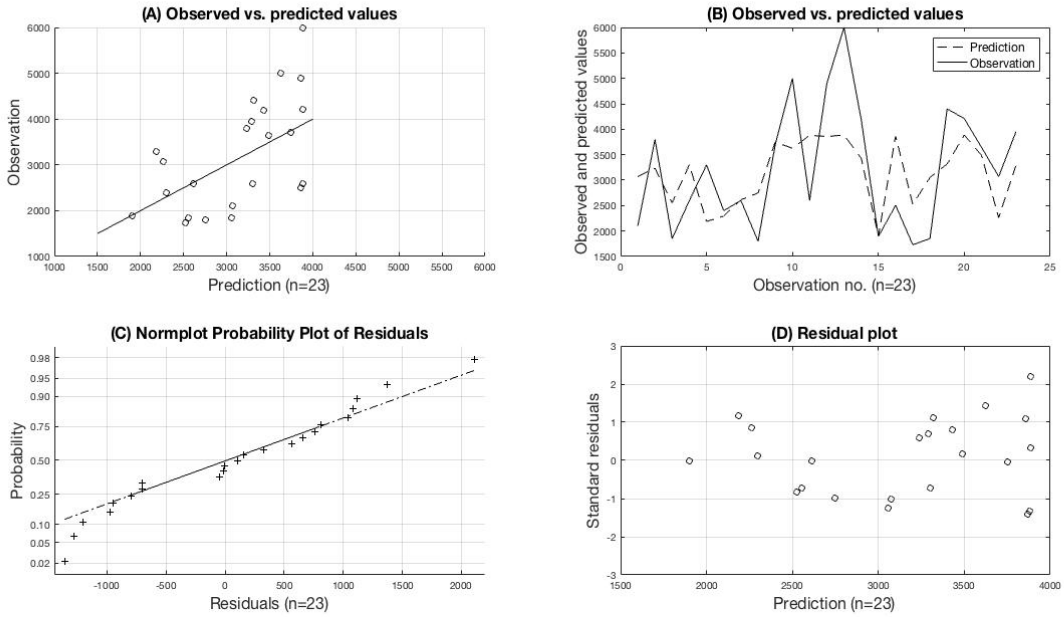


Fig. 5. Goodness-of-fit illustration of the first model element for coarse soils

Shapiro-Wilk normality test: W = 0.95798, p-value = 0.4238

Paired t-test: t = t = -0.44954, df = 22, p-value = 0.6574

Figure 5a shows that there are observations that the model overestimates and the variance of the observations increases slightly for higher observations (Fig.5a, b and d). Thus, there is some heteroskedasticity present. Figure 5c shows, however, that the distribution of the residuals is not significantly different from a normal distribution. We hypothesise that the primary reason for the heteroscedasticity is the cap within the data set; there is notably more data points for low STP levels than for high STP levels. This generates heteroscedasticity because the variance is considerable (for low STP values and most likely for high STP levels also).

2. Model element (Square Root/Quadratic)

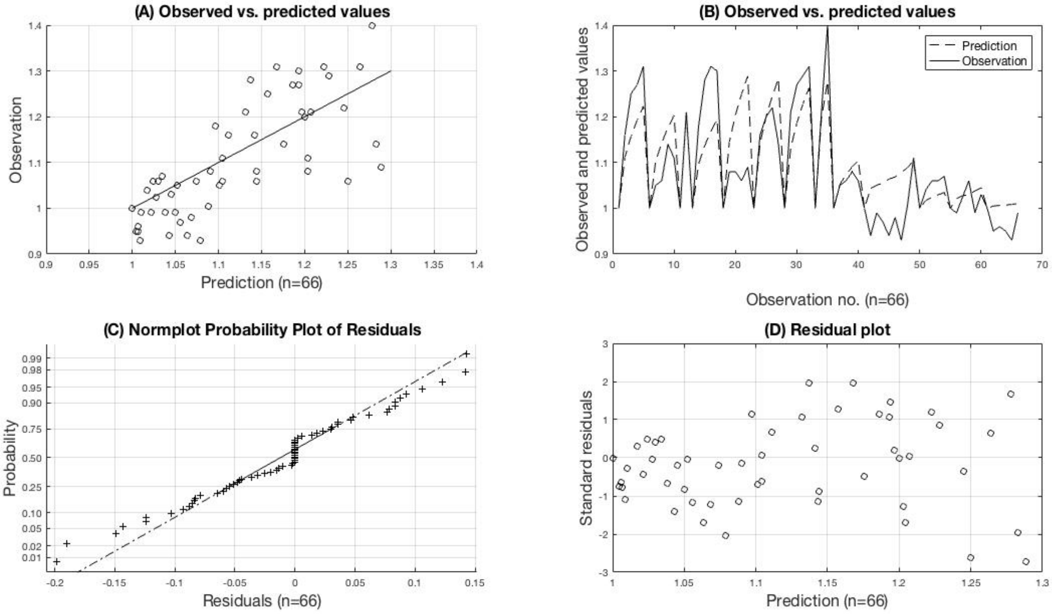


Fig. 6. Goodness-of-fit illustration of the second model element for coarse soils

Shapiro-Wilk normality test: W = 0.97025, p-value = 0.1135

Paired t-test: t = 1.0543, df = 65, p-value = 0.2957

Figure 6c indicates that the residuals are not significantly different from the normal distribution, but Figures 6b, 6c and 6d show that the model almost systematically overestimates some of the observations. In addition, variation of the residuals as well as the residuals appears to increase for higher predictions. We may notice that the model does not explain well the variation within the observations 40-60. However, there is no reason to exclude these observations from the sample. Further, the overall fit and the normality of the residuals got worse if the constant is changed from 1 to minimum response. Thus, the model element is the best fit to sample although there is clearly some amount of heteroscedasticity within the model error residuals. This is supported also by the pared t-test since we do not reject the H_0 of difference between predicted and observed means being zero. The observations 40-60 were related to the experiments where there was high STP level and as a result there was a high variation within the P response. This finding is in agreement with results by Griffith (1992), according to which there are many factors that may affect the response to P fertilisation on soils with high STP levels.

In general, heteroscedasticity within the model error residuals might be a natural phenomenon; since for small fertiliser amounts applied the yield is likely to be small in any case, whereas for larger fertiliser amounts applied, the seasonal variability has a greater effect on annual yields. Also Upadhyay et al. (2006) noted that yield data for fertiliser is often heteroscedastic with higher yield variability associated with higher fertiliser rates. The heteroscedasticity remained, although the short-term experiments were less weighted in estimation. We hypothesise, that to reduce the amount of heteroscedasticity, we could introduce weather-related variables to the model in order to take into account the relationship between seasonal variability and increasing fertilisation rates. In addition, longer time-series for field observations might eliminate the heteroscedasticity, since the averaged observations are the less affected by the annual weather related variation the longer the experiments are.

3. Model element (Logarithm/Quadratic)

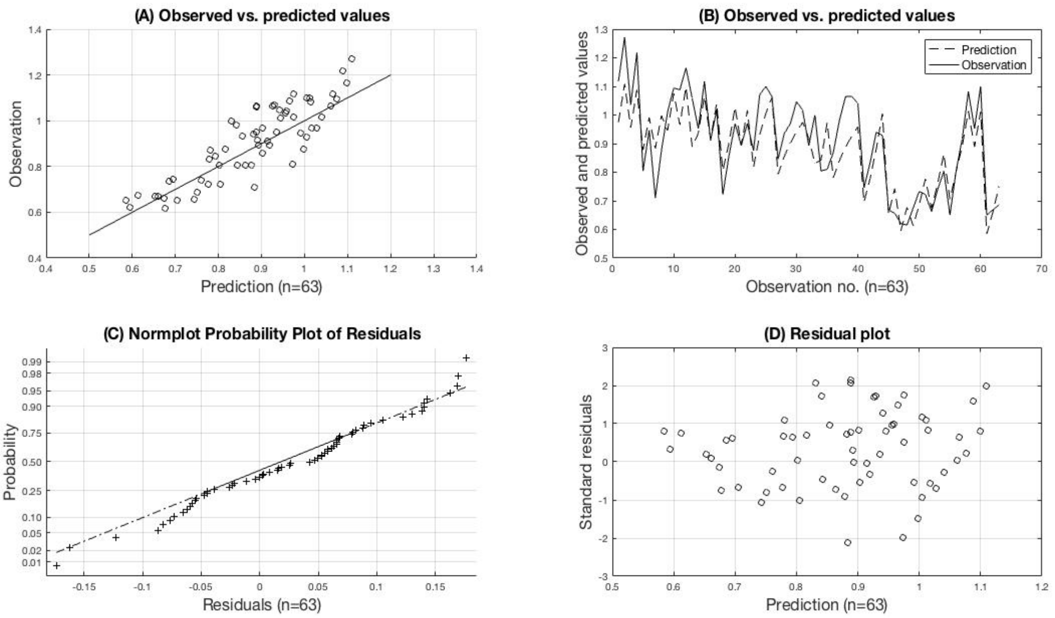


Fig. 7. Goodness-of-fit illustration of the third model element for coarse soils

Shapiro-Wilk normality test: W = 0.97673, p-value = 0.2768

Paired t-test: t = 0.685, df = 62, p-value = 0.4959

Figure 7 shows that the residuals are normally distributed and there is no heteroscedasticity present. In addition, model clearly predicts the observation patterns accurately.

Appendix 5: Analytical properties of the system models

5.1. Analytical properties of the transition models

The partial derivative of the transition function with respect to P fertilisation is the following:

$$\frac{\partial \Delta STP_t}{\partial P_t} = (\delta_2 + \delta_3 STP_t)\left(1 - \frac{\beta_1}{STP_t}\frac{\partial STP_t}{\partial P_t}Y_t - (\beta_1 \log(STP_t) + \beta_2)\frac{\partial Y_t}{\partial P_t}\right) \tag{1}$$

Since $\frac{\partial Y_t}{\partial P_t} > 0$, $\frac{\partial \Delta STP_t}{\partial P_t} > 0$ only if $1 > \frac{\beta_1}{STP_t}\frac{\partial STP_t}{\partial P_t}Y_t + (\beta_1 \log(STP_t) + \beta_2)\frac{\partial Y_t}{\partial P_t}$, which implies that P fertilisation has a positive effect on change in STP only if the direct marginal effect of P exceeds the indirect effect via crop-uptake, i.e. if the P balance is positive. The partial derivative of the transition function with respect to the STP is the following:

$$\frac{\partial \Delta STP_t}{\partial STP_t} = -(\delta_2 + \delta_3 STP_t)\left(\frac{\beta_1}{STP_t} Y_t + (\beta_1 \log(STP_t) + \beta_2)\left[\left(\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} + y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}\right)\omega_{N,t}\right]\right) + \delta_3(P_t - (\beta_1 \log(STP_t) + \beta_2)Y_t) - \delta_4. \quad (2)$$

Since $\frac{\partial y_{P0,t}}{\partial STP_t} > 0$ and $\frac{\partial \omega_{P,t}}{\partial STP_t} < 0$ we have following conditions:

(i) when $\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} > y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}$, then $\frac{\partial \Delta STP_t}{\partial STP_t} > 0$ only if $\delta_3 P_t > (\delta_2 + \delta_3 STP_t)\left(\frac{\beta_1}{STP_t} Y_t + (\beta_1 \log(STP_t) + \beta_2)\left[\left(\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} + y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}\right)\omega_{N,t}\right]\right) + \delta_3(\beta_1 \log(STP_t) + \beta_2)Y_t + \delta_4$

(ii) when $\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} < y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}$, then $\frac{\partial \Delta STP_t}{\partial STP_t} > 0$ only if $\delta_3 P_t - (\delta_2 + \delta_3 STP_t)\left(\frac{\beta_1}{STP_t} Y_t + (\beta_1 \log(STP_t) + \beta_2)\left[\left(\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} + y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}\right)\omega_{N,t}\right]\right) > \delta_3(\beta_1 \log(STP_t) + \beta_2)Y_t + \delta_4$

Condition (i) implies that if the positive marginal effect of STP dominates the negative marginal effect on yield $\left(\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} > y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}\right)$, the annual STP will accumulate the STP stock if annual effect of P fertilisation exceeds the marginal effect of STP on crop-uptake, i.e. if the annual P-balance is positive. Condition (ii) implies that if the negative marginal effect of STP dominates the positive marginal effect on yield $\left(\frac{\partial y_{P0,t}}{\partial STP_t}\omega_{P,t} < y_{P0,t}\frac{\partial \omega_{P,t}}{\partial STP_t}\right)$, the STP will accumulate the STP stock if annual effect of P fertilisation and the marginal effect of STP on crop-uptake exceed the absolute amount of P that is removed with the yield.

The partial derivate of the transition function with respect to N fertilisation is the following:

$$\frac{\partial \Delta STP_t}{\partial N_t} = (\delta_2 + \delta_3 STP_t)\left(-\frac{\beta_1}{STP_t}\frac{\partial STP_t}{\partial N_t} Y_t - (\beta_1 \log(STP_t) + \beta_2)\frac{\partial Y_t}{\partial N_t}\right) \quad (3)$$

Since $\frac{\partial Y_t}{\partial N_t} > 0$ and $\frac{\partial STP_t}{\partial N_t} < 0$ we have a following condition:

(i) if $\frac{Y_t}{STP_t} > \left(\log(STP_t) + \frac{\beta_2}{\beta_1}\right)\frac{\partial Y_t}{\partial N_t}\left(\frac{\partial STP_t}{\partial N_t}\right)^{-1}$, then $\frac{\partial STP\Delta_t}{\partial N_t} > 0$

(ii) if $\frac{Y_t}{STP_t} < \left(\log(STP_t) + \frac{\beta_2}{\beta_1}\right)\frac{\partial Y_t}{\partial N_t}\left(\frac{\partial STP_t}{\partial N_t}\right)^{-1}$, then $\frac{\partial STP\Delta_t}{\partial N_t} < 0$

Since the phenomenon is complicated, we examined the effects of the STP, P and N fertilisation and P balance on transition function numerically. Figure 8 illustrates the relationships between the transition function and its arguments. Figure 8 shows that STP transition was an increasing function of P fertilisation, STP and P-balance whereas the transition function was a decreasing function of N fertilisation. The STP transition appeared to be a convex function of P fertilisation and P-balance.

The relationship between STP and STP transition was almost linear. Figure 8 also shows that the relationships differed for different soil textures, although these differences most likely originated from the differences in yield responses. The interaction of N, P and STP, as well as the dynamic behaviour of the system described in this research originated from these properties.

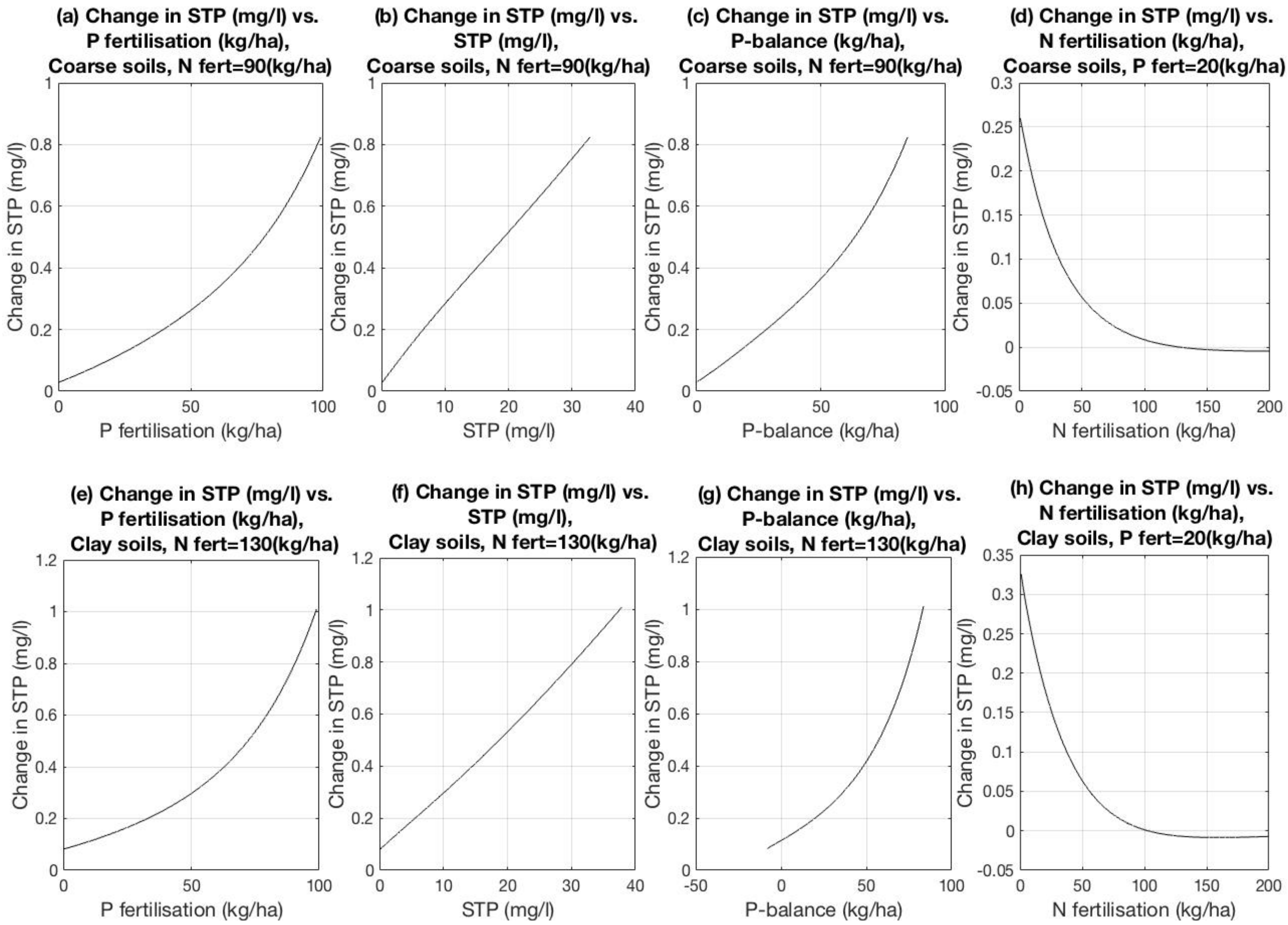


Fig. 8. The effect of P fertilisation, STP, P-balance and N fertilisation on STP transition function on coarse and clay soils

Appendix 6: Analytical properties of the estimated yield response models

6.1. Marginal productivity of the P and N fertilisers and the STP

The marginal products of the inputs are described with the following partial derivatives (where $\theta_{l,j}$ indicates the parameter with the subscript $l \in [1,3]$ denoting a model element, and subscript j denoting a parameter number):

$$(MP)_{N,coarse} = \frac{\partial \bar{Y}_{coarse}}{\partial N} = Y_{P0}\big(\theta_{1,j}, STP\big)\omega_P\big(\theta_{2,j}, P, STP\big)\frac{\frac{2\theta_{3,2}log(N+1)}{(N+1)}}{\left(1+\theta_{3,2}{Y_{N0}}^2\right)} > 0 \qquad (4)$$

$$\frac{\partial^2 \bar{Y}_{coarse}}{\partial N^2} = -Y_{P0}\big(\theta_{1,j}, STP\big)\omega_P\big(\theta_{2,j}, P, STP\big)\frac{\frac{2\theta_{3,2}log(N+1)}{(N+1)^2}}{\left(1+\theta_{3,2}{Y_{N0}}^2\right)} < 0 \qquad (5)$$

$$(MP)_{P,coarse} = \frac{\partial \bar{Y}_{coarse}}{\partial P} = Y_{P0}\left(\theta_{1,j}, STP\right)\left(\frac{\frac{1}{2}\theta_{2,1}P^{-\frac{1}{2}}}{1+\theta_{2,2}STP^2}\right)\omega_N\left(\theta_{3,j}, N, Y_{N0}\right) > 0 \qquad (6)$$

$$\frac{\partial^2 \bar{Y}_{coarse}}{\partial P^2} = -\frac{1}{4}Y_{P0}\left(\theta_{1,j}, STP\right)\left(\frac{\theta_{2,1}P^{-\frac{3}{2}}}{1+\theta_{2,2}STP^2}\right)\omega_N\left(\theta_{3,j}, N, Y_{N0}\right) < 0 \qquad (7)$$

The equations 4 and 6 show that the marginal products of both inputs are positive. The equations 5 and 7 show that both inputs exhibit diminishing marginal products.

The partial derivative with respect to STP is more complex:

$$(MP)_{STP,coarse} = \frac{\partial \bar{Y}_{coarse}}{\partial STP} = \left\{\frac{STP^{\frac{1}{\theta_{1,1}}-1} + \theta_{1,2}STP^{\theta_{1,3}+\frac{1}{\theta_{1,1}}-1}\gamma}{\left(1+\theta_{1,2}STP^{\theta_{1,3}}\right)^2}\left(\frac{\theta_{2,1}\sqrt{P}}{1+\theta_{2,2}STP^2} + 1\right) - \right.$$

$$\left.\frac{2\theta_{2,2}\theta_{2,1}\sqrt{P}}{\left(1+\theta_{2,2}STP^2\right)^2}\frac{\theta_{1,1}STP^{\frac{1}{\theta_1}+1}}{\left(1+\theta_{1,2}STP^{\theta_{1,3}}\right)}\right\}\omega_N\left(\theta_{3,j}, N_i, Y_{N0}\right),$$

where $\gamma = 1 - \theta_{1,1}\theta_{1,2}\theta_{1,3} > 0$ (8)

In this case we have the following condition:

$$\frac{STP^{\frac{1}{\theta_{1,1}}-1} + \theta_{1,2}STP^{\theta_{1,3}+\frac{1}{\theta_{1,1}}-1}\gamma}{\left(1+\theta_{1,2}STP^{\theta_{1,3}}\right)^2}\left(\frac{\theta_{2,1}\sqrt{P}}{1+\theta_{2,2}STP^2} + 1\right) \underset{<}{\geq} \frac{2\theta_{2,2}\theta_{2,1}\sqrt{P}}{\left(1+\theta_{2,2}STP^2\right)^2}\frac{\theta_{1,1}STP^{\frac{1}{\theta_1}+1}}{\left(1+\theta_{1,2}STP^{\theta_{1,3}}\right)} \rightarrow \frac{\partial \bar{Y}_{coarse}}{\partial STP} \underset{<}{\geq} 0$$

The realised parameter values and the point in STP/P-space determine which case holds.

The marginal products for clay soils are the following partial derivatives:

$$(MP)_{N,clay} = \frac{\partial \bar{Y}_{clay}}{\partial N} = Y_{P0}\left(\theta_{1,j}, STP\right)\omega_P\left(\theta_{2,j}, P, STP\right)\frac{\frac{1}{2\sqrt{N}}\theta_{3,1}}{\theta_{3,3}Y_{N0}{}^2+1} > 0 \qquad (9)$$

$$\frac{\partial^2 \bar{Y}_{clay}}{\partial N^2} = -\frac{1}{4}Y_{P0}\left(\theta_{1,j}, STP\right)\omega_P\left(\theta_{2,j}, P, STP\right)\frac{N^{-\frac{3}{2}}\theta_{3,1}}{\theta_{3,3}Y_{N0}{}^2+1} < 0 \qquad (10)$$

$$(MP)_{P,clay} = \frac{\partial \bar{Y}_{clay}}{\partial P} = Y_{P0}\left(\theta_{1,j}, STP\right)\left(\frac{\frac{1}{2}\theta_{2,1}P^{-\frac{1}{2}}}{1+e^{\theta_{2,3}STP}}\right)\omega_N\left(\theta_{3,j}, N, Y_{N0}\right) > 0 \qquad (11)$$

$$\frac{\partial^2 \bar{Y}_{clay}}{\partial P^2} = -\frac{1}{4}Y_{P0}\left(\theta_{1,j}, STP\right)\left(\frac{\theta_{2,1}P^{-\frac{3}{2}}}{1+e^{\theta_{2,3}STP}}\right)\omega_N\left(\theta_{3,j}, N, Y_{N0}\right) < 0 \qquad (12)$$

Since the second derivatives are negative, it is clear that the assumption of diminishing marginal productivity (the concavity of the yield response function with respect to the inputs) holds.

In the case of clay soils the partial derivative with respect to STP is the following:

$$(MP)_{STP,clay} = \frac{\partial \bar{Y}_{clay}}{\partial STP} = \left\{ \frac{\theta_1\theta_2 STP^{\theta_2-1}(1+\sqrt{STP}) - \theta_1(STP^{\theta_2}+1)\frac{1}{2\sqrt{STP}}}{(1+\sqrt{STP})^2}\left(\frac{\theta_{2,1}\sqrt{P}+\theta_{2,2}}{1+e^{\theta_{2,3}STP}} + \omega_{P,min}\right) - \frac{\theta_1(STP^{\theta_2}+1)}{(1+\sqrt{STP})}\left(\frac{(\theta_{2,1}\sqrt{P}+\theta_{2,2})\theta_{2,3}e^{\theta_{2,3}STP}}{(1+e^{\theta_{2,3}STP})^2}\right)\right\}\omega_N(\theta_{3,j},N,Y_{N0}) \qquad (13)$$

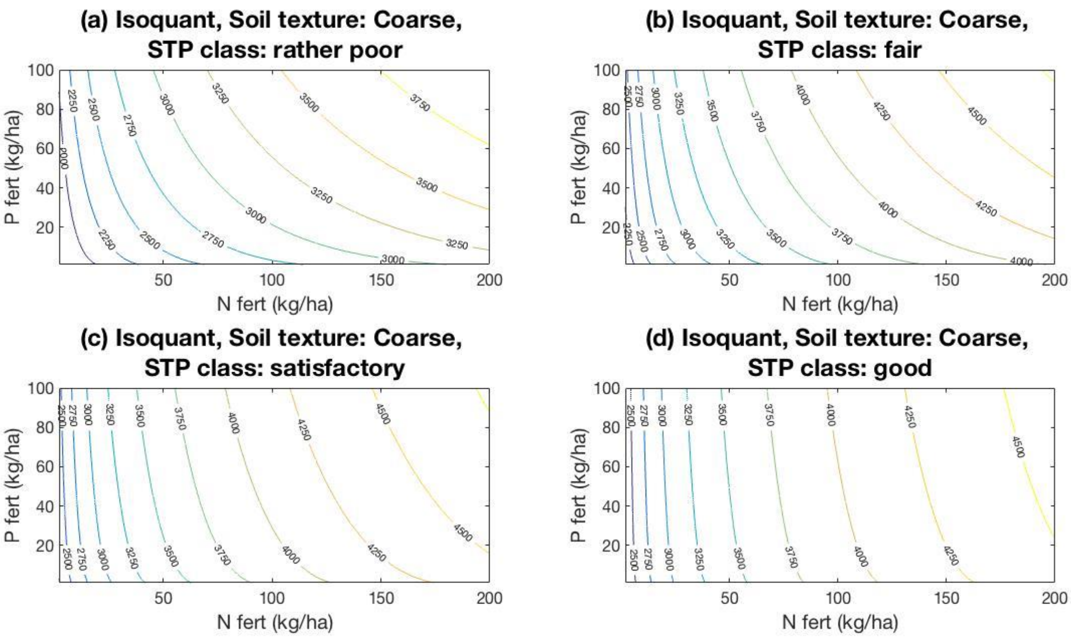
In this case we have the following condition:

$$\frac{\theta_1\theta_2 STP^{\theta_2-1}(1+\sqrt{STP}) - \theta_1(STP^{\theta_2}+1)\frac{1}{2\sqrt{STP}}}{(1+\sqrt{STP})^2}\left(\frac{\theta_{2,1}\sqrt{P}+\theta_{2,2}}{1+e^{\theta_{2,3}STP}} + \omega_{P,min}\right) \gtrless \frac{\theta_1(STP^{\theta_2}+1)}{(1+\sqrt{STP})}\left(\frac{(\theta_{2,1}\sqrt{P}+\theta_{2,2})\theta_{2,3}e^{\theta_{2,3}STP}}{(1+e^{\theta_{2,3}STP})^2}\right) \rightarrow \frac{\partial \bar{Y}_{coarse}}{\partial STP} \gtrless 0$$

Again the realised parameter values determine which case holds.

6.2. Isoquants

We may examine the quasiconcavity of the models with Figure 9, which represents the isoquants (input vectors producing the same level of output: $Q(y) = \{\boldsymbol{x} \in R^n : y = f(\boldsymbol{x})\}$ or simply the level curves of the underlying production function) of the models for various fixed STP levels.



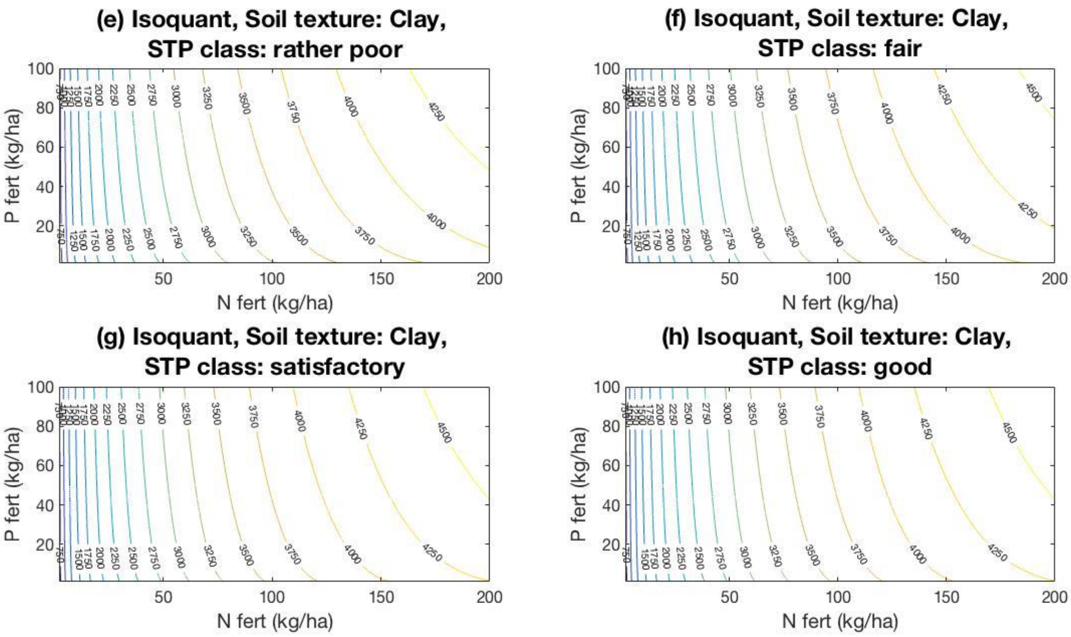


Fig. 9. Isoquants for both soil textures for various STP levels (STP level rather poor is 3 mg l^{-1} for clay and 5 mg l^{-1} for coarse soils, STP level fair is 6 mg l^{-1} for clay and 10 mg l^{-1} for coarse soils, STP level satisfactory is 11.5 mg l^{-1} for clay and 17.5 mg l^{-1} for coarse soils and STP level good is 20 mg l^{-1} for clay and 28.5 mg l^{-1} for coarse soils)

The isoquants for both soil textures are clearly convex. Therefore the models are quasiconcave with respect to N and P. Thus the following inequality holds: $y(\theta N + (1-\theta)P) \geq min\big(y(N), y(P)\big)$for any combination of N and P and $\theta \in [0,1]$. The quasiconcavity of the models with respect to N and P inputs implies that the inputs are complements to each other to some degree; both inputs are important for production. It becomes apparent from the Figure 9, however, that the relative importance of the P fertilisation becomes lower for higher STP level. From Figure 9 it becomes apparent that it is required more P fertilisation to compensate the decrease in N fertilisation in the case of clay soils than in the case of coarse soils in order to produce the same amount of yield, i.e. to stay on the same isoquant. Thus, the N is relatively more important factor of production for clay soils.

6.3. Elasticity of the yield with respect to the P and N fertilisers (*partial output elasticity*)

The elasticity of the yield with respected to N and P for coarse soils may be described with the following equations:

$$\epsilon_{N,coarse} = \frac{\partial \bar{Y}_{coarse}}{\partial N} \frac{N}{\bar{Y}_{coarse}} = \frac{2\theta_{3,2} log(N+1)}{\theta_{3,1} + \theta_{3,2}(log(N+1))^2} \frac{N}{(N+1)} \tag{14}$$

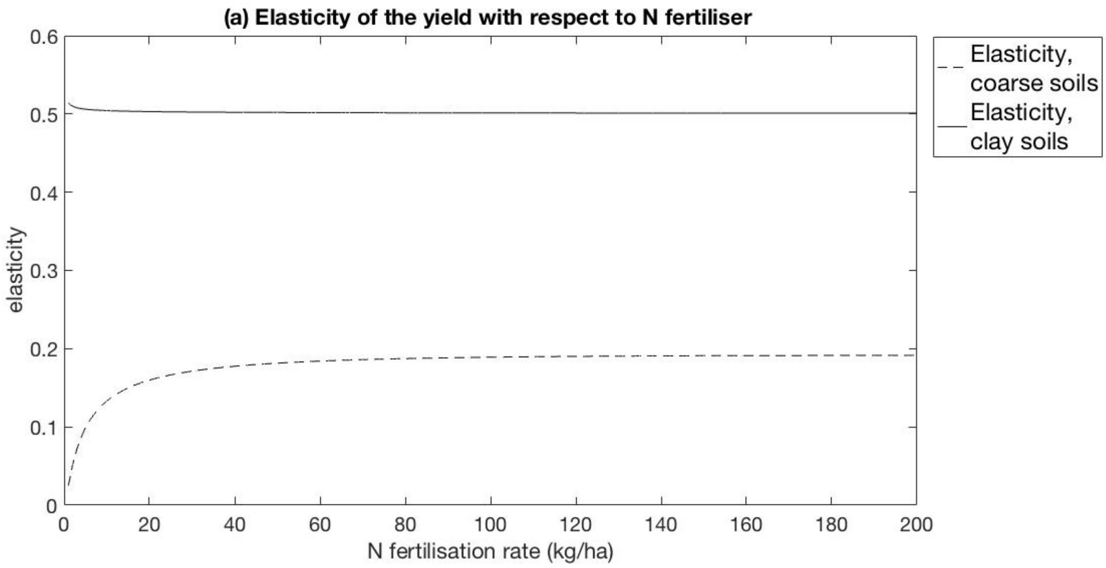
$$\epsilon_{P,coarse} = \frac{\partial \bar{Y}_{coarse}}{\partial P} \frac{P}{\bar{Y}_{coarse}} = \frac{\frac{1}{2}\theta_{2,1}\sqrt{P}}{\theta_{2,1}\sqrt{P} + 1 + \theta_{2,2} STP^2} \tag{15}$$

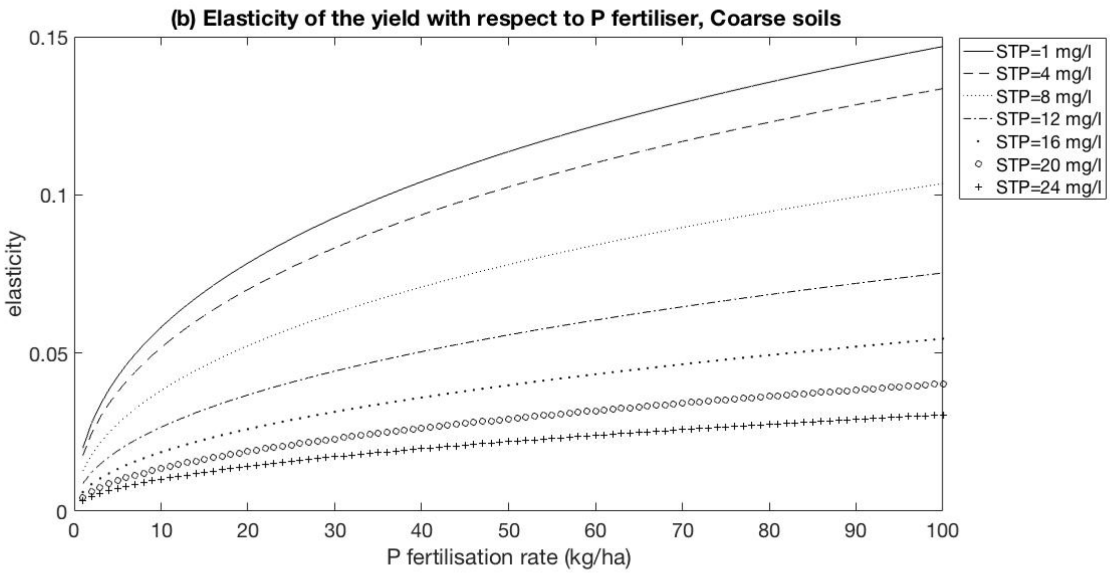
$$\epsilon_{N,clay} = \frac{\partial \bar{Y}_{clay}}{\partial N} \frac{N}{\bar{Y}_{clay}} = \frac{\frac{1}{2}\theta_{3,1}\sqrt{N}}{\theta_{3,1}\sqrt{N}+\theta_{3,2}} \tag{16}$$

$$\epsilon_{P,clay} = \frac{\partial \bar{Y}_{clay}}{\partial P} \frac{P}{\bar{Y}_{clay}} = \frac{\frac{1}{2}\theta_{2,1}\sqrt{P}}{\theta_{2,1}\sqrt{P}+\theta_{2,2}+\omega_{P,min}\left(1+e^{\theta_{2,3}STP}\right)} \tag{17}$$

It must be noted that for neither soil texture does the output elasticity with respect to N depend on the N control yield. Instead, the output elasticity with respect to P depends on the STP level. This results from the fact that there is a constant part in a second model element. Because of the constant part the effect of the STP does not cancel out.

Figure 10 illustrates these elasticity measures. From Figure 10 it becomes apparent that $\epsilon_{N,coarse} < \epsilon_{N,clay}$ and $\epsilon_{P,coarse} > \epsilon_{P,clay}$. This would suggest that the output for clay soils is more influenced by the fluctuations in N input applications while the output for coarse soils is more influenced by the fluctuations in P input applications.





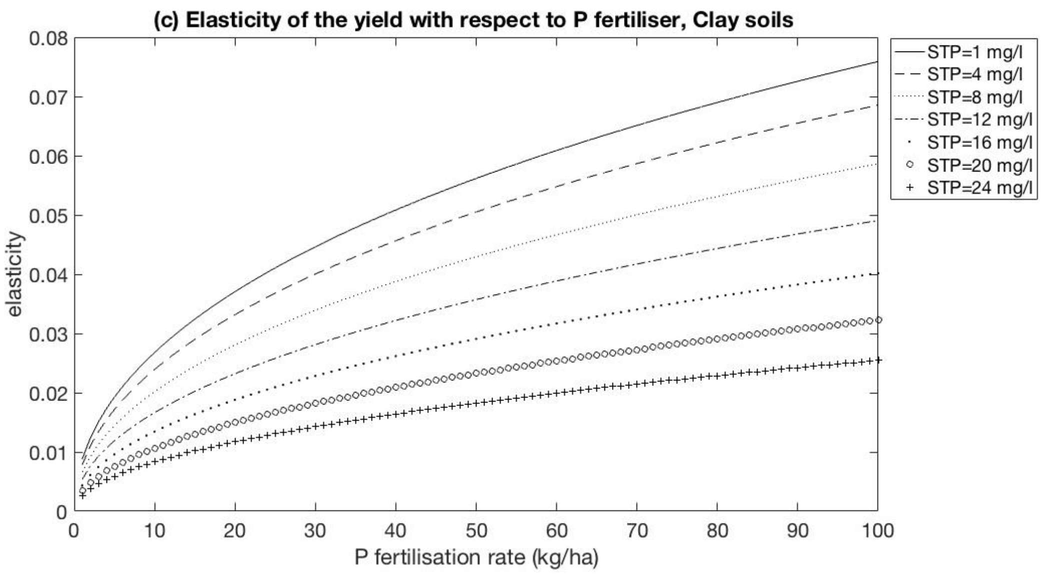


Fig. 10. Elasticity of the yield with respect to the P and N fertilisers measured at different points

Appendix 7: Results of the Monte Carlo analysis

We obtained the yield distributions for the optimal input rates and STP levels via the Monte Carlo analysis. The mean of the yield distribution was different from the yield obtained within the optimisation, although the input vector was the same: for coarse soils the mean of the distribution was 1.7% lower than the yield obtained in optimisation, whereas for clay soils the mean of the distribution was 3% higher than the yield obtained in optimisation (Table 2). Thus, the difference was not notable, and it stemmed from the fact that the yield distribution was not perfectly symmetric (Fig. 11). For the coarse soils, the mean was 0.27% lower than the median, which indicated that the distribution was left-skewed (skewness=-0.233), although the distribution was fairly symmetrical, which was verified by Shapiro-Wilk normality test (p-value=7.069e-13). In contrast, for the clay soils, the mean was 4% higher than the median, which indicated that the distribution was moderately rightskewed (skewness=0.624), although the Shapiro-Wilk normality test indicated that there was no strong evidence against the normality of the distribution (p-value=2.2e-16).

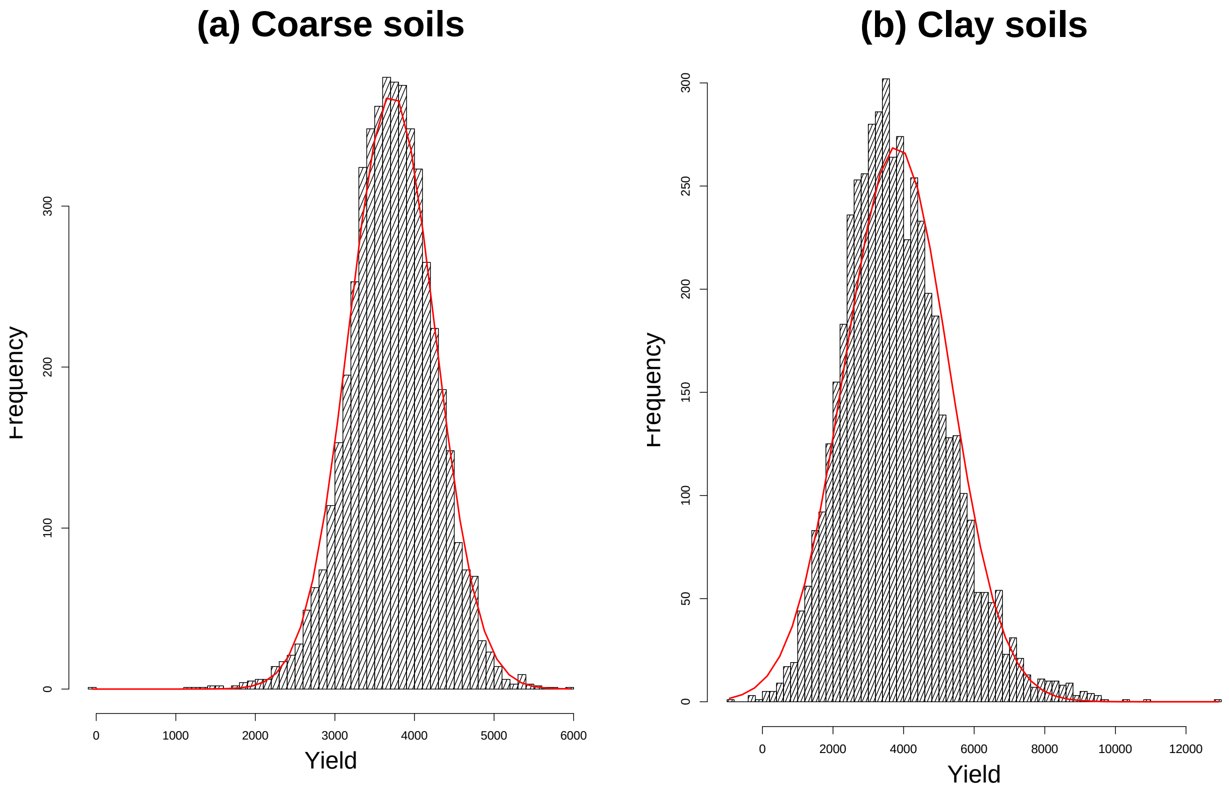


Fig. 11. The distributions for optimal yields for coarse (a) and clay (b) soils when parametric uncertainty relates to all the model parameters

Table 2 verifies the results obtained by comparing the structural uncertainty and the parametric uncertainty by DSA, according to which the parameter uncertainty was greater for clay soils models, since the CV was 164% higher for clay soils than for coarse soils when all the model parameters were considered. In addition, most of the parameter uncertainty was related to the first model element in the case of coarse soils models, whereas most of the parameter uncertainty was related to third model element in the case of clay soils, as was noticed above. Particularly in the case of clay soils, the parametric uncertainty was related almost entirely to the third model element. By comparing the means and medians of the optimal yields, it is also observable that the most uncertain model elements were also primary causes for the skewness of the distributions (Table 2).

Table 2
Summary statistics of the yield distributions

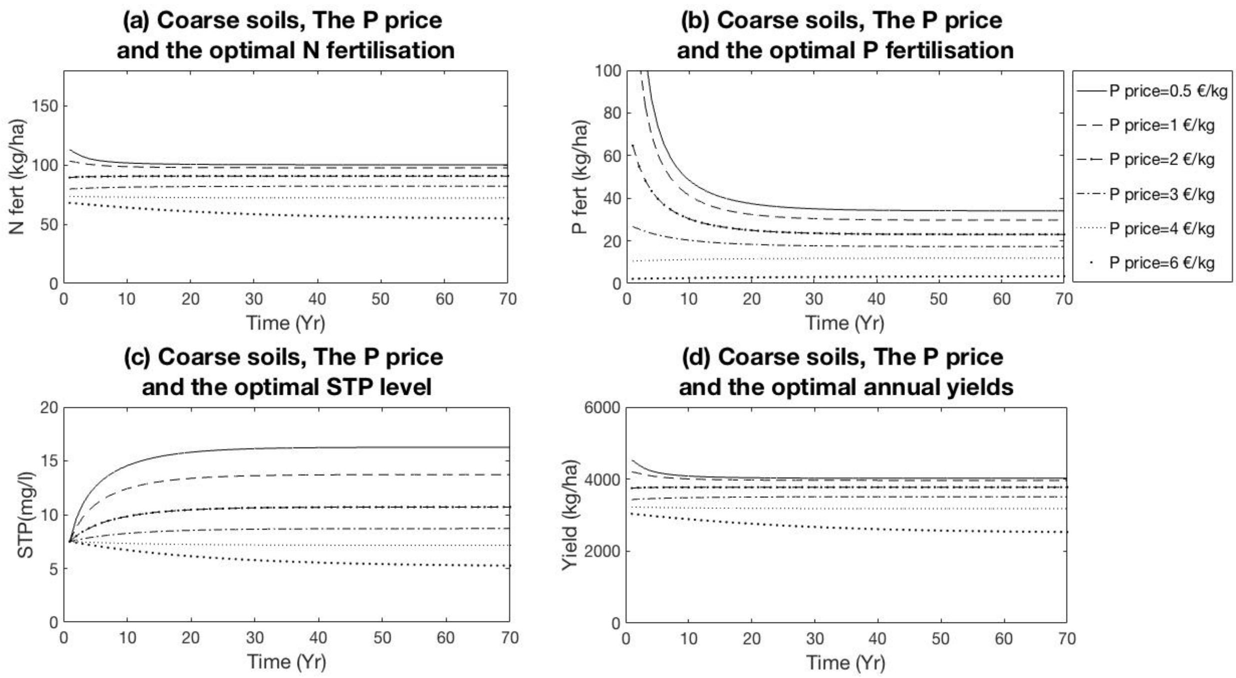
Soil texture	Parameters considered	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	SD	CV
Coarse	1.model element	210	3410	3720	3698	4018	5403	473	12.8%
Coarse	2.model element	3577	3731	3781	3797	3843	5272	99	2.6%
Coarse	3.model element	2918	3605	3771	3774	3938	4692	244	6.5%
Coarse	All the parameters	85.34	3367	3727	3717	4081	6081	552	14.8%
Clay	1.model element	2604	3475	3690	3693	3903	4956	319	8.6%

Clay	2.model element	3412	3649	3693	3694	3738	3945	66	1.8%
Clay	3.model element	0	2833	3721	3821	4685	11770	1439	38%
Clay	All the parameters	0	2806	3655	3800	4663	14450	1483	39%

Appendix 8: Results of the sensitivity analysis

Since in the obtained optimums the economic parameters as well as the N-control yields and the initial STP levels were fixed to certain values, it was of the interest to examine the effect of these parameters on the economic optimums. Therefore, we applied partial sensitivity analysis in order to examine the effect of these parameters on economic optimums one parameter at the time while holding all the other parameters fixed.

Figure 12 shows that the P price had a considerable effect on optimal P fertilisation and as a result a moderate effect on STP level. Instead, the P price had relatively minor effect on optimal N fertilisation. Since the N fertilisation was more important factor in determining the annual yield than the P fertilisation, the effect of the P price on optimal annual yields was relatively minor. Nevertheless, for every variable, the higher the P price, lower was the absolute amount of the variable, which would suggest that P and N are not easily substituted by each other; both inputs are important factors of production. The P price had a greater effect on P fertilisation and STP level for clay soils compared to coarse soils (Fig. 12b, c, f and g). Instead, the effect on N fertilisation and annual yield was greater for coarse soils (Fig. 12a, d, e and h). This would suggest that the demand for P fertilisation was more elastic for clay soils, whereas the output elasticity of P was higher for coarse soils.



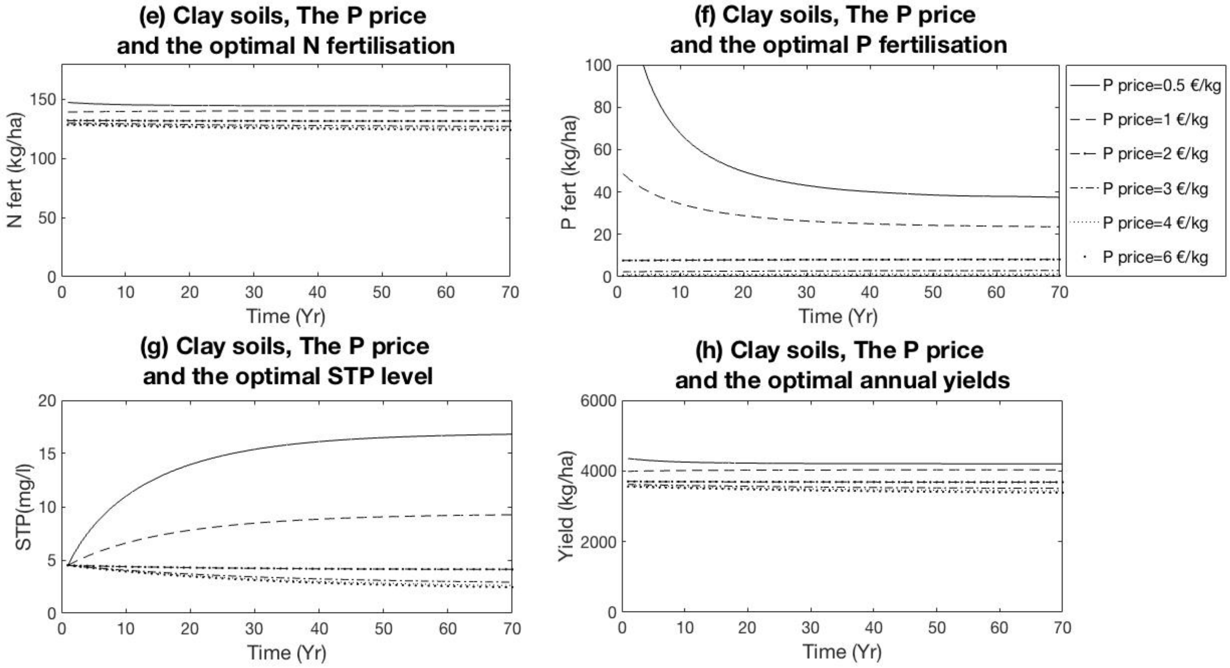
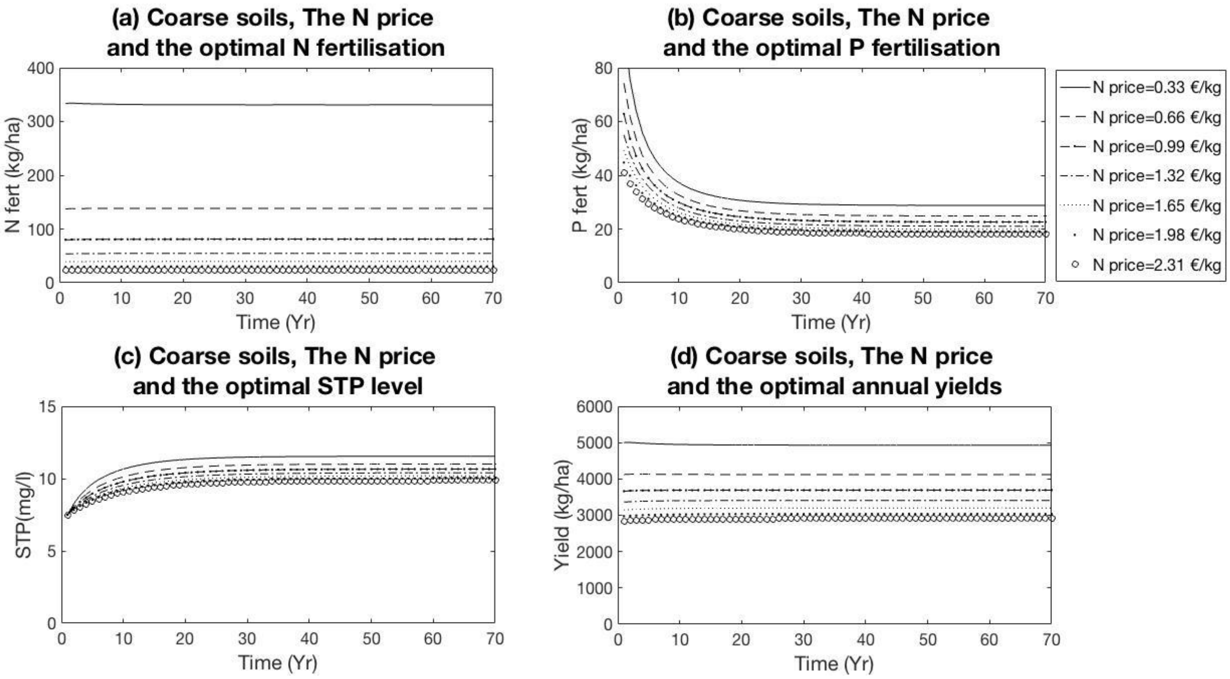


Fig. 12. The sensitivity of the optimisation results with respect to the P prices

Figure 13 shows that the N price had a great effect on optimal N fertilisation rate for both soil textures. As a result, the effect of N price was great also on the annual yields (because the N was important factor of production). Instead, the effect of N price was relatively minor on optimal STP level (Fig. 13c and g). This reflected the fact that the N fertilisation affected the STP only indirectly through crop-uptake. The effect of N price was similar for all the variables: the higher the N price, the lower was the optimal level of the variable. Since the demand for P was more elastic for clay soils, adjusting the P fertilisation in the case of clay soils could compensate the effects of price fluctuations in N fertilisation on N demand to some degree, although the N was relatively more important factor for clay soils and the output elasticity of N is higher for clay soils. In addition, the demand for N was more elastic for coarse soils.



(a) Coarse soils, The N price and the optimal N fertilisation
N fert (kg/ha)
Time (Yr)
(b) Coarse soils, The N price and the optimal P fertilisation
P fert (kg/ha)
Time (Yr)
N price=0.33 €/kg
N price=0.66 €/kg
N price=0.99 €/kg
N price=1.32 €/kg
N price=1.65 €/kg
N price=1.98 €/kg
N price=2.31 €/kg
(c) Coarse soils, The N price and the optimal STP level
STP(mg/l)
Time (Yr)
(d) Coarse soils, The N price and the optimal annual yields
Yield (kg/ha)
Time (Yr)

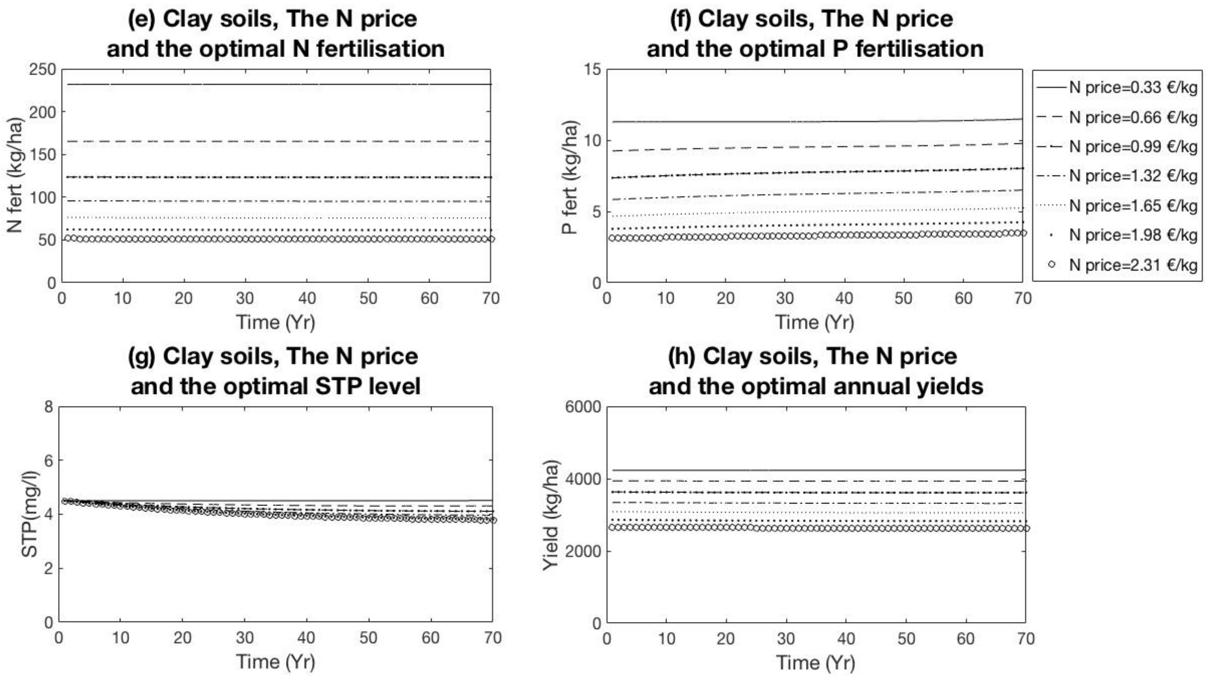
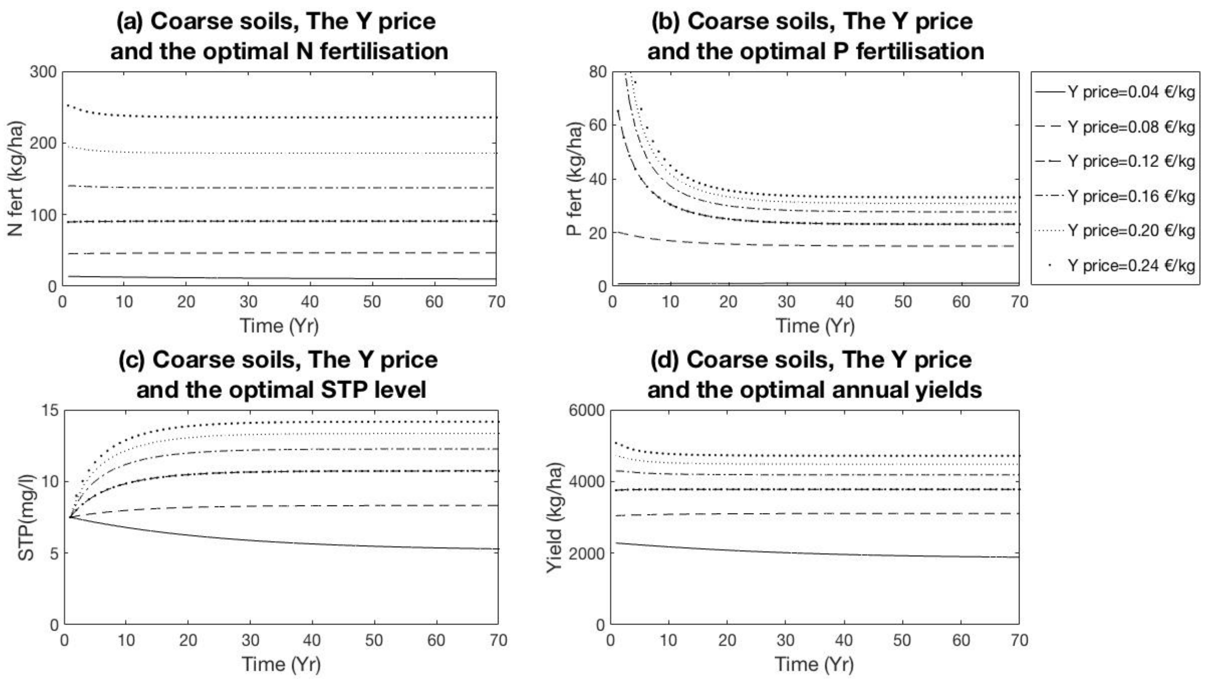


Fig. 13. The sensitivity of the optimisation results with respect to the N prices

Figure 14 illustrates the effect of barley yield price on economic optimums: the higher was the yield price, the higher was the optimal fertilisation rate, STP and the annual yield. It became also apparent that the effect of the barley yield price on yield was greater for coarse soils compared to clay soils indicating greater elasticity for yield price for coarse soils.



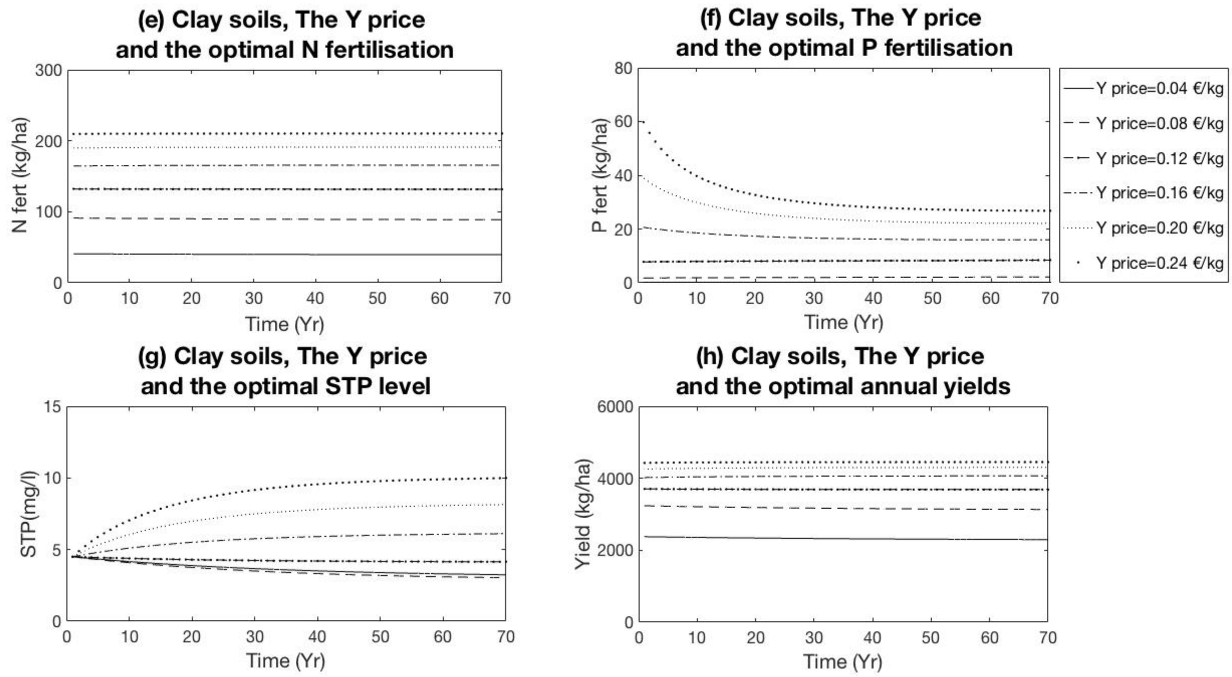
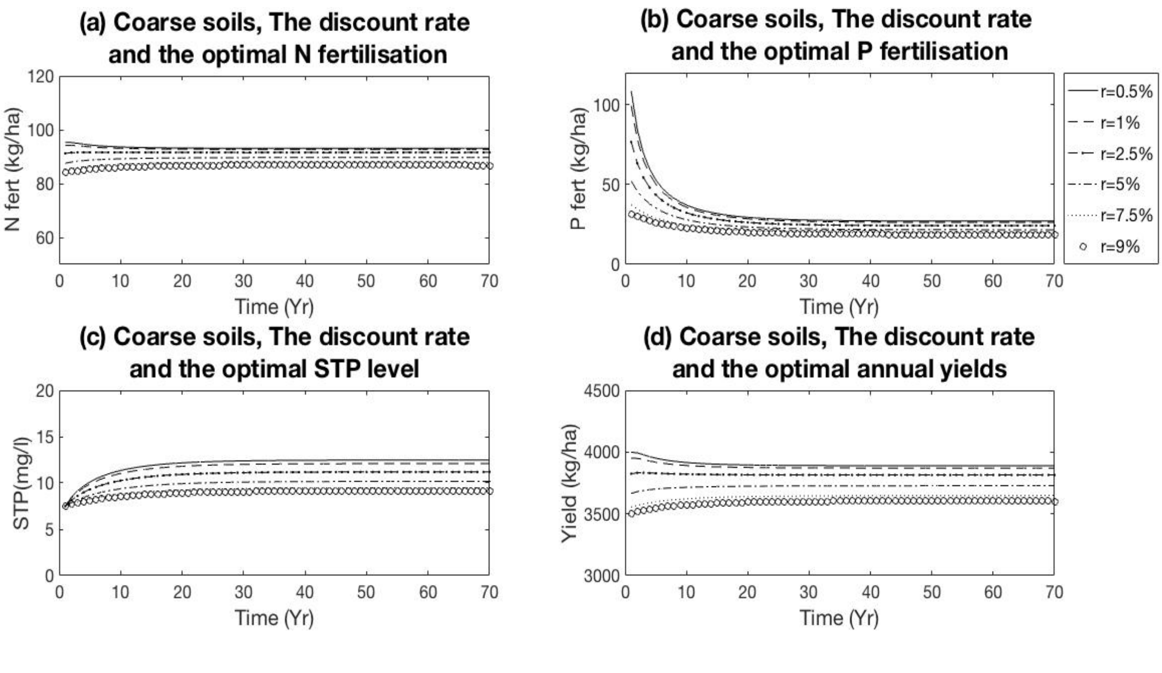


Fig. 14. The sensitivity of the optimisation results with respect to the Y prices

Figure 15 illustrates the effect of the discount rate on economic optimums. Figure 15 shows that the higher was the discount rate, the lower were the optimal fertilisation rates and as a result the optimal STP levels and the optimal annual yields. Nevertheless, the effect of a discount rate was relatively minor compared to the other examined parameters. The discount rate affects particularly the most dynamic elements of the system model, i.e. the STP and P fertilisation. Figure 15 shows that the effect of the discount rate on P rate and STP is greater for clay soils, which again reflects the higher elasticity for P demand for clay soils. As a result, the effect of discount rate on annual yields is also greater for clay soils.



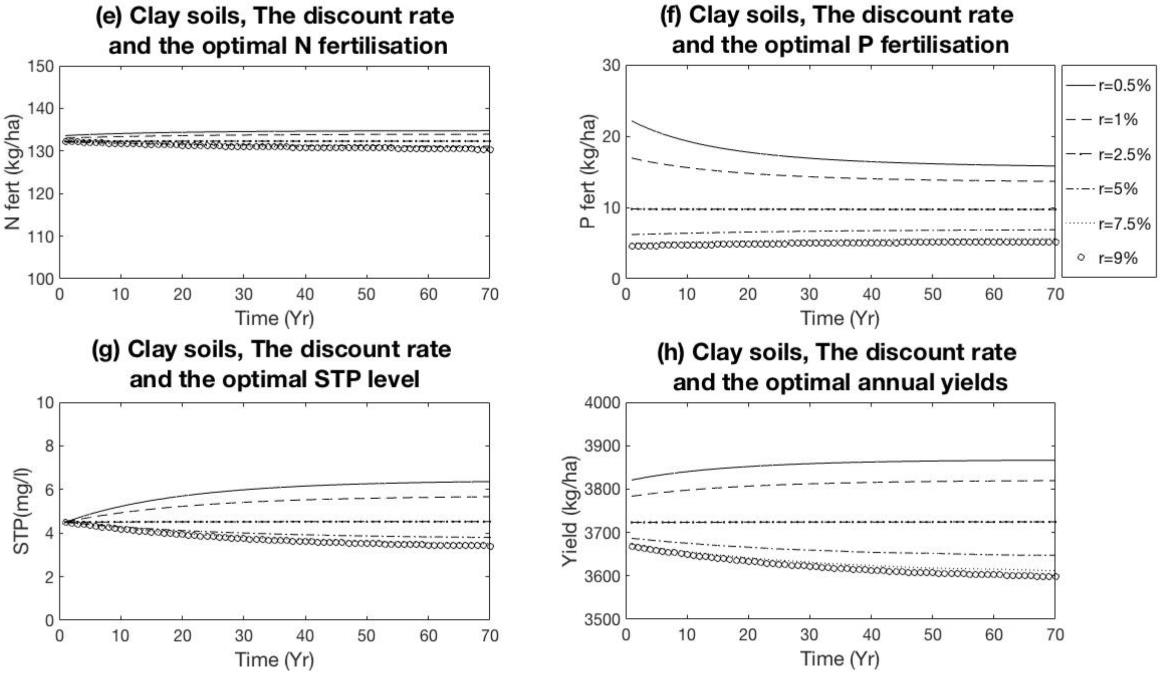


Fig. 15. The sensitivity of the optimisation results with respect to the discount rate.

Figure 16 illustrates the effect of the initial STP level. Figure 16 shows that despite the initial STP level, the steady state optimum was always the same when all the other parameters remained unchanged; the initial STP level had no effect on the steady state optimum. However, the initial STP level affected the optimal path of the variable: when initial STP was considerably low, it was optimal to apply considerably high P rate and low N rate in the beginning of the planning horizon. As a result, the N and P fertilisation rates, STP level and annual yield settled fast to their optimal steady state paths. The explanation of such behaviour is that since $\frac{\partial STP_{t+1}}{\partial N_t} = \frac{\partial \vartheta}{\partial y_t}\frac{\partial y_t}{\partial N_t} < 0$ and $\frac{\partial STP_{t+1}}{\partial P_t} = \frac{\partial \vartheta}{\partial P_t} + \frac{\partial \vartheta}{\partial y_t}\frac{\partial y_t}{\partial P_t} > 0$ if $\frac{\partial \vartheta}{\partial P_t} > \frac{\partial \vartheta}{\partial y_t}\frac{\partial y_t}{\partial P_t}$, it is optimal to maintain low crop-uptake in the beginning of the planning horizon by applying a low dose of N fertiliser as well as to apply a high dose of P fertiliser in order to get the STP to its optimal steady state path fast. Once the STP is at its optimal level it is also optimal to settle the N and P rates to their optimal levels. Instead, when the initial STP level is considerably high, there is no need for P fertilisation for a long time and as a result the optimal P fertilisation rate rises steadily to its optimal steady state level (Fig.16.b, f). Correspondingly the optimal N level is high at the beginning of the planning horizon and it lowers to its steady state level steadily (Fig.16.a, e).

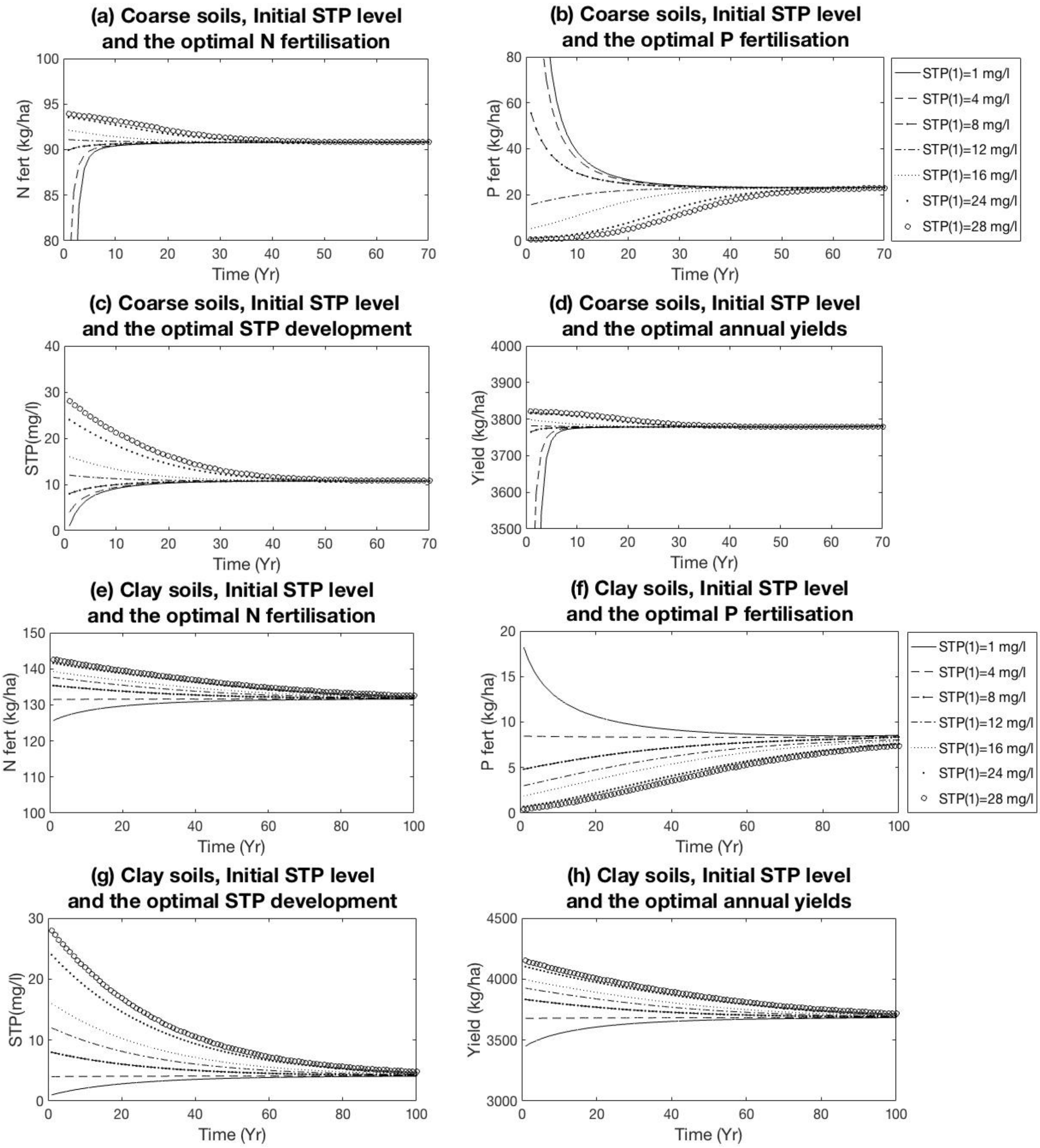


Fig. 16. The sensitivity of the optimisation results with respect to the initial STP level.

Last we examined the effect of the N control yield-parameter on economic optimums. Figure 17 shows that the N control yield had a considerable effect on optimal fertilisation rates, particularly on optimal N fertilisation rate, and the optimal annual yield. The effect of N control yield was greater for clay soils compared to that for coarse soils. This again resulted from the fact that for clay soils the N was relatively more important factor than for coarse soils. The N control yield decreased the yield increasing effect of the N fertilisation. For clay soil this effect was more crucial than for coarse soils because the output elasticity of N was higher for clay soils.

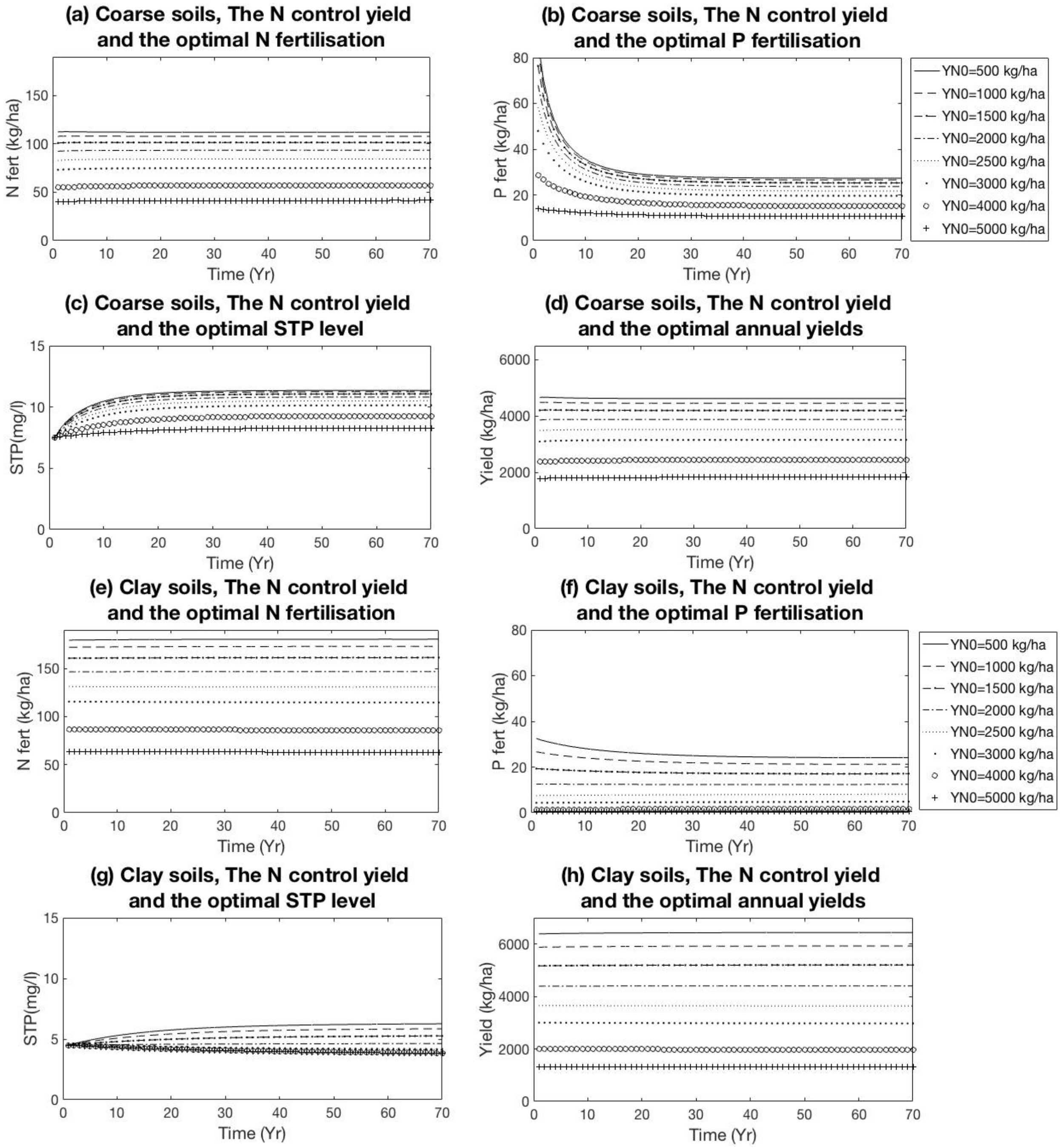


Fig. 17. The sensitivity of the optimisation results with respect to the N control yield

Appendix 9: References for Appendix

Akaike, H., (1973): Information theory as an extension of the maximum likelihood principle. Pages 267-281 in B.N. Petrov and F. Csaki, editors. Second International Symposium on Information Theory. Akademiai Kiado, Budapest.

Burnham, K.P., and Anderson, D.R., (2002): Model Selection and Multimodel Inference: A practical information-theoretic approach. Second Edition. Springer

Busman L., Lamb J., Randall G., Rehm G-, and Schmitt M., (2002). The nature of phosphorus in soils. Copyright © 2002 Regents of the University of Minnesota. All rights reserved. Reviewed July 2009

Griffin R.C., Montgomery J.M., and Rister M.E., (1987): Selecting Functional Form in Production Function Analysis. Western Journal of Agricultural Economics, 12(2): 216-227.

Griffith, W.K., (1992): Crop responses at high soil test phosphorus levels. Better Crops/Fail 1992

Jensen L.S., and Schjoerring J.K., (2011). Benefits of nitrogen for food, fiber and industrial production. The European Nitrogen Assessment, ed. Mark A. Sutton, Clare M. Howard, Jan Willem Erisman, Gilles Billen, Albert Bleeker, Peringe Grennfelt, Hans van Grinsven and Bruna Grizzetti. Published by Cambridge University Press. ©

Cambridge University Press 2011, with sections © authors/European Union. Available at the address: http://www.nine-esf.org/sites/nine-esf.org/files/ena_doc/ENA_pdfs/ENA_c3.pdf
Saarela I., Järvi A., Hakkola H. & Rinne K., (1995): Fosforilannoituksen porraskokeet 1977 – 1994. Vuosittain annetun fosforimäärän vaikutus maan viljavuuteen ja peltokasvien satoon monivuotisissa kenttäkokeissa. MAATALOUDEN TUTKIMUSKESKUS TIEDOTE 16/95
Shapiro, S. S., and Wilk, M. B., (1965): "Analysis of variance test for normality (complete samples)", Biometrika 52: 591–611.)
Tumusiime, E., Brorsen, B.W., Mosali, J., Johnson, J., Locke, J., and Biermacher, J.T., (2011): Determining Optimal Levels of Nitrogen Fertiliser Using Random Parameter Models. Journal of Agricultural and Applied Economics, 43,4(November 2011): 541-552
Upadhyay, B.M., Smith, E.G., and Eavret, M.L. (2006): An Innovative Approach for Modeling Crop Yield Response to Fertiliser Nutrients. Selected Paper , CEA-CAES annual meeting, Montreal, Quebec, May 25-28, 2006

Supplementary Material: Analytical data

The data utilised for estimation of the response models is presented in Tables 1-8.

Table 1. Dataset for first model element for clay soils

STP	P-control Yield	weight
0.8	3970	3
4.0	3300	6
5.0	2000	3
2.7	3500	4
5.9	3800	5
6.4	4700	5
40.0	4200	5
20.5	4300	5
10.0	3200	6
2.8	3304	18
5.3	3068	12
11.3	2905	12
43.7	4571	15
50.5	4565	15
7.2	3777	18
3.5	3391	12
8.5	3333	11
4.6	4749	15

Table 2. Dataset for second model element for clay soils

P-fertilisation	STP	Yield response	weight
0	0.8	1.000	3
6	0.8	1.070	3
25	0.8	1.060	3
50	0.8	1.140	3
100	0.8	1.160	3
0	4.0	1.000	6
15	4.0	1.100	6
30	4.0	1.060	6
45	4.0	1.170	6
60	4.0	1.190	6

0	5.0	1.000	3
18	5.0	1.050	3
0	2.7	1.000	4
18	2.7	1.080	4
36	2.7	1.130	4
0	5.9	1.000	5
8	5.9	1.010	5
13	5.9	1.060	5
15	5.9	1.070	5
25	5.9	1.030	5
35	5.9	1.020	5
0	6.4	1.000	5
8	6.4	1.010	5
13	6.4	1.010	5
15	6.4	1.170	5
25	6.40	1.040	5
35	6.4	1.170	5
0	40.0	1.000	5
15	40.0	1.000	5
30	40.0	1.020	5
45	40.0	1.020	5
60	40.0	0.990	5
0	20.5	1.000	5
8	20.5	1.010	5
13	20.5	0.970	5
25	20.5	0.960	5
35	20.5	1.080	5
0	10.0	1.000	6
15	10.0	1.010	6
30	10.0	1.030	6
45	10.0	0.994	6
60	10.0	1.047	6

Table 3. Dataset for third model element for clay soils. Notice that the yield response is scaled N response utilised in estimation.

N fertilisation	N-control yield	Yield response	weight
50	1800	0.8058424	2
75	1800	0.9649578	2
100	1800	1.0573474	2
125	1800	1.0624801	2
150	1800	1.0676129	2
54	1700	0.7339839	2
108	1700	0.8161079	2
54	1900	0.7442494	2
50	2800	0.6261960	3
75	2800	0.6518598	3

100	2800	0.6210633	3
125	2800	0.6723908	3
150	2800	0.6313288	3
50	2450	0.8161079	6
75	2450	0.9495596	6
100	2450	1.0419491	6
125	2450	1.0368164	6
150	2450	1.0573474	6
30	2200	0.7853114	4
60	2200	0.9957543	4
90	2200	1.1292060	4
120	2200	1.2472593	4
150	2200	1.3345161	4
180	2200	1.3858437	4
90	2800	0.8263734	3
90	3000	0.8161079	3
50	2100	0.7545149	3
75	2100	0.9598251	3
100	2100	1.0522146	3
125	2100	1.1086749	3
150	2100	1.1446042	3
50	2400	0.5851340	2
75	2400	0.6621253	2
100	2400	0.7237184	2
125	2400	0.8109752	2
150	2400	0.8058424	2

Table 4. Dataset for first model element for coarse soils

STP	P-control Yield	weight
7.0	2100	10
8.0	3800	6
4.801	1850	4
8.501	2600	4
3.701	3300	4
4.0	2400	4
5.0	2600	7
5.501	1800	3
15.0	3700	16
12.101	5000	6
30.0	2600	8
21.901	4900	5
49.8	6000	3
9.6	4189	16
3.0	1893	15

23.5	2507	12
4.7	1730	16
6.9	1853	12
8.6	4401	9
41.1	4209	12
10.2	3645	18
3.9	3070	12
8.4	3951	9

Table 5. Dataset for second model element for coarse soils

P fertilisation	STP	Yield response	weight
0	7.0	1.000	10
15	7.0	1.160	10
30	7.0	1.250	10
45	7.0	1.270	10
60	7.0	1.310	10
0	8.0	1.000	6
15	8.0	1.050	6
30	8.0	1.060	6
45	8.0	1.140	6
60	8.0	1.110	6
0	4.8	1.000	4
36	4.8	1.210	4
0	8.5	1.000	4
15	8.5	1.180	4
30	8.5	1.280	4
45	8.5	1.310	5
60	8.5	1.300	5
0	3.7	1.000	4
15	3.7	1.080	4
30	3.7	1.080	4
45	3.7	1.060	4
60	3.7	1.090	4
0	4.0	1.000	4
15	4.0	1.160	4
30	4.0	1.200	4
45	4.0	1.220	4
60	4.0	1.140	4
0	5.0	1.000	7
15	5.0	1.210	7
30	5.0	1.270	7
45	5.0	1.290	7
60	5.0	1.310	7
0	5.5	1.000	3
36	5.5	1.210	3
72	5.5	1.400	3

0	15.0	1.000	16
15	15.0	1.050	16
30	15.0	1.060	16
45	15.0	1.080	16
60	15.0	1.060	16
0	12.1	1.000	6
6	12.1	0.940	6
8	12.1	0.990	6
10	12.1	0.970	6
13	12.1	0.940	6
15	12.1	0.980	6
20	12.1	0.930	6
25	12.1	1.005	6
35	12.1	1.110	6
0	30.0	1.000	8
15	30.0	1.040	8
30	30.0	1.060	8
45	30.0	1.060	8
60	30.0	1.070	8
0	21.9	1.000	5
8	21.9	0.990	5
13	21.9	1.025	5
15	21.9	1.060	5
25	21.9	0.990	5
35	21.9	1.030	5
0	49.8	1.000	3
8	49.8	0.950	3
13	49.8	0.960	3
15	49.8	0.950	3
25	49.8	0.930	3
35	49.8	0.990	3

Table 6. Dataset for third model element for coarse soils. Notice that the yield response is scaled N response utilised in estimation.

N fertilisation	N control yield	Yield response	weight
54	1300	1.0759752	2
108	1300	1.2239929	2
54	1500	0.9962733	2
108	1500	1.1727560	2
26	1000	0.7742467	3
52	1000	0.9108785	3
27	1000	0.6831588	2
54	1000	0.8425626	2
54	1610	0.9735013	2
108	1610	1.0532032	2
54	1400	1.0475102	3

108	1400	1.1215191	3
40	1680	1.0190453	4
54	1700	0.9165714	2
108	1700	1.0759752	2
54	1900	0.8767205	2
108	1900	0.9791943	2
54	2800	0.6945448	2
54	2000	0.8254836	3
108	2000	0.9336504	3
54	2080	0.8596415	3
108	2080	0.9336504	3
54	2700	0.8425626	5
108	2700	1.0304313	5
162	2700	1.0588962	5
216	2700	1.0247383	5
50	2800	0.8140976	3
75	2800	0.8994925	3
100	2800	0.9336504	3
125	2800	1.0076593	3
150	2800	0.9791943	3
80	2400	0.8596415	3
50	2500	0.9621154	2
50	2400	0.7742467	2
108	2400	0.7799397	2
50	2900	0.8368696	3
75	2900	0.9450364	3
100	2900	1.0247383	3
125	2900	1.0247383	3
150	2900	1.0019663	3
16	2400	0.7173168	6
32	2400	0.8027116	6
64	2400	0.9051855	6
128	2400	0.8937995	6
54	4000	0.6433079	2
108	4000	0.6319219	2
54	4500	0.5977640	2
108	4500	0.5920710	2
16	3200	0.6490009	3
32	3200	0.7059308	3
64	3200	0.6945448	3
26	3100	0.6376149	2
52	3100	0.7116238	2
103	3100	0.7742467	2
50	3500	0.6262289	2
100	3500	0.7742467	2
54	2100	0.8824135	5

108	2100	1.0418172	5
54	2120	0.9165714	5
108	2120	1.0588962	5
27	3940	0.6262289	4
54	3940	0.6433079	4
108	3940	0.6603869	4

Table 7. Validation dataset for coarse soils

N fertilisation	P fertilisation	STP	Yield	weight
0.00	0.0000	12.500000	1223.333	3
50.00	13.0000	12.500000	2986.667	3
100.00	26.0000	12.500000	3810.000	3
0.00	0.0000	21.600000	1978.333	5
50.00	9.4000001	21.600000	3162.000	5
100.00	18.0000	21.600000	4237.500	5
80.00	17.0000	18.700000	3508.750	2
160.00	34.0000	18.700000	3558.750	2
0.00	0.0000	20.500000	2892.500	4
60.00	13.1000	20.500000	4140.000	4
80.00	17.4000	20.500000	4350.000	4
100.00	21.8000	20.500000	4387.500	4
120.00	26.2000	20.500000	4570.000	4
140.00	30.5000	20.500000	4627.500	4
160.00	34.9000	20.500000	4567.500	4
0.00	0.0000	20.692857	2396.000	5
48.00	9.20000001	20.692857	3338.000	5
64.00	12.20000001	20.692857	3452.000	5
80.00	15.3000	20.692857	3618.000	5
96.00	18.3000	20.692857	4195.000	5
65.00	14.2000	20.692857	4070.000	5
78.00	17.0000	20.692857	4117.500	5
79.90	12.29520001	6.592593	3605.000	1
80.00	6.9760001	6.592593	3630.000	1
80.50	10.68200001	6.592593	3465.000	1
80.00	5.58080001	6.592593	3720.000	1
80.00	6.97600001	6.592593	3675.000	1
130.05	20.0124	6.592593	4065.000	1
130.00	11.336000001	6.592593	3945.000	1
129.95	17.2438	6.592593	3885.000	1
130.00	9.0688	6.592593	3905.000	1
45.00	11.80000001	8.400000	4350.000	1
90.00	23.5000	8.400000	4850.000	1
49.50	35.9700	3.900000	2300.000	1
100.50	73.0300	3.900000	3670.000	1

Table 8. Validation dataset for clay soils

N fertilisation	P fertilisation	STP	Yield	weight
0.00	0.0000	4.400000	940.000	3
100.00	13.3000	4.400000	3106.667	3
100.00	8.700000001	4.400000	3200.000	3
100.00	19.0000	4.400000	3083.333	3
99.00	19.2000	4.400000	3330.000	3
171.90	0.0000	4.400000	2783.333	3
0.00	0.0000	9.400000	1410.000	1
100.00	13.3000	9.400000	4170.000	1
100.00	8.70	9.400000	4560.000	1
100.00	19.0000	9.400000	4420.000	1
99.00	19.2000	9.400000	4420.000	1
171.90	0.0000	9.400000	4470.000	1
0.00	0.0000	9.800000	1015.000	3
100.00	13.3000	9.800000	3165.000	3
100.00	8.70	9.800000	3340.000	3
100.00	19.0000	9.800000	3430.000	3
99.00	19.2000	9.800000	3315.000	3
171.90	0.0000	9.800000	3035.000	3
0.00	0.0000	18.200000	894.000	10
50.00	11.0000	18.200000	2970.000	10
100.00	22.0000	18.200000	4159.200	10
150.00	33.0000	18.200000	4892.000	10
200.00	44.0000	18.200000	5194.000	10
0.00	0.0000	12.400000	1231.000	10
50.00	11.0000	12.400000	2927.000	10
100.00	22.0000	12.400000	3693.000	10
150.00	33.0000	12.400000	4182.000	10
200.00	44.0000	12.400000	4316.000	10
52.80	10.1000	7.738889	2380.000	6
88.00	16.8000	7.738889	3146.667	6
158.40	30.2000	7.738889	3435.000	6
48.00	9.1560	11.900000	2695.000	6
72.00	13.7000	11.900000	3426.667	6
96.00	18.3000	11.900000	3410.000	6
0.00	0.0000	8.400000	1713.000	2
104.00	24.0000	8.400000	3595.000	2
104.00	23.0000	8.400000	3825.000	2
105.00	46.0000	8.400000	3800.000	2
105.00	76.0000	8.400000	3785.000	2
60.00	13.0800	31.200000	4907.500	1
120.00	26.1600	31.200000	4842.500	1
52.50	22.9000	20.000000	3740.000	1
105.00	45.8000	20.000000	4030.000	1

150.00	65.4000	20.000000	3680.000	1
60.00	11.445	16.643750	1884.286	7
120.00	22.8900	16.643750	2000.000	7
50.25	13.1454	24.300000	3957.500	2
100.50	26.2908	24.300000	4347.500	2
150.00	39.2400	24.300000	4256.250	2
80.00	17.4400	16.900000	4158.000	2
160.00	34.8800	16.900000	4203.000	2
0.00	0.0000	4.600000	1709.000	3
50.25	36.5150	4.600000	2773.667	3
100.50	73.0300	4.600000	3429.000	3
150.00	109.0000	4.600000	3706.833	3
0.00	0.0000	3.200000	2811.667	3
90.00	52.3200	3.200000	4508.333	3
180.00	104.6400	3.200000	4575.000	3
60.00	43.6000	10.766667	4007.111	3
120.00	87.2000	10.766667	4421.389	3
60.00	43.6000	10.600000	2220.000	1
120.00	87.2000	10.600000	2600.000	1
40.05	23.2824	11.200000	2100.000	1
80.10	46.5648	11.200000	3376.667	1
0.00	0.0000	8.350000	1833.333	3
100.00	58.1624	8.350000	3504.444	3
75.00	43.6000	8.600000	1845.667	10
150.00	87.2000	8.600000	2591.000	10